Continuous Multistate System Universal Generating Function

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Abstract

Usually, systems and components are described as being in one of two modes, "on" or "off." Such systems are described using binary structure functions. In multistate systems (MSS), components can be in more than two states—for example, there can be partially failed or partially operating modes. The system state can be described by continuously many values. A system that can have different task performance levels is named multi-state system (MSS). In this paper, we present a technique for solving a family of Continuous MSS reliability problems. A universal generating function (UGF) method is proposed for fast reliability estimation of continuous MSSs. The UGF method provides the ability to estimate relatively quickly different MSS reliability indices for series-parallel, parallel-series and bridge structures. It can be applied to MSS with different physical nature of system performance measure.

Keywords: multi-state system, universal generating function, reliability

I. Introduction

Many technical systems are distinguished by their structural complexity. They can perform their task at several different levels like the system failure can lead to decreased ability to perform the given task without moving to complete failure. Such a system element can also perform its task with some different levels in between perfect functioning to complete failure. Such systems are named as Multistate Systems (MSS). For example, the generating unit in power systems has its nominal generating capacity, which is fully available if there are no failures. Some types of failures can cause complete unit outage while other types of failures can cause a unit to work with reduced capacity. For example, in a power generation system, the generator can produce 100MW, later on, due to some technical problem, it may produce 80MW and so on. The physical characteristics of the performance depend on physical nature of the system outcome. One need to choose reliability procedures according to the physical performance of system such as productivity, capacity etc. Therefore, it is important to measure performance rates of system components by their contribution into the entire MSS output performance. Continuous materials or energy transmission systems or oil transportation systems, power generation systems etc are examples of MSS. Billinton and Allan (1996), Aven (1990) discussed the flow network problem, which provide the desired throughput or transmission capacity for continuous energy, material or information flow. There may be components in series or parallel in the system. In redundancy optimization problems, data processing speed can also be considered as a performance measure and the main task of system is to complete the task within the desired time, see Levitin et.al (1998) and Lisnianski et.al (2000). Several type of MSS were considered in Gnedenko and Ushakov (1995). Several properties of MSSs and importance measures in MSSs are considered in Chacko and Manoharan (2008, 2011).

A rigorous work on Binary state system can be seen in Barlow and Proschan (1975). Many standard works on reliability theory adopt a framework in which systems and components can be in only one of two modes 'on' or 'off'. Consequently, the system structure function is a binary function of binary variables. Most of the standard results in Barlow and Proschan (1975) are set in this framework. These structure functions fail to model important situations when systems have redundant standby components. Furthermore, if the components or systems can be in intermediate modes besides the two extremes of completely functioning or completely failed, the above framework does not suffice. To remedy this situation, authors such as Barlow and Wu (1978), El. Neweihi et al. (1978), and Griffith (1980) have considered situations in which components and systems can assume a finite number of values. In these works, the basic concepts of MSS reliability were formulated and the system structure function was defined for coherent MSS. The research on such systems – called 'multi-state systems' is still continuing. The aim of this research work is to advance the state-of-the art of the highly promising multi-state reliability theory so that it can be applied to design and maintenance of practical engineering systems. In Griffith (1980), the coherence definition was generalized and three types of coherence were studied. The reliability importance was extended to MSS from the binary state system in Butler (1979). Concepts of MSS importance are also discussed in Block and Savits (1982).

The steady state behavior of Markov systems is very useful in reliability analysis. The steady-state behavior of multi-state monotone systems was considered by applying the theory for stationary and synchronous processes with embedded point process in Natvig and Streler (1984). The modeling technique was suggested by Wood (1985), which allows existing binary algorithms for block diagrams and fault trees to be applied to multi-state system. The concept of equivalent behavior was introduced in Garriba et.al (1980) which provide the analysis of multiple-valued logic tree aimed at eliciting prime implicants. These prime implicants are the multiple-valued logic analogue of minimal cut sets encountered in binary fault trees. The prime implicants were also successfully used in dependability analysis of software controlled systems Yau (1998). As in the Binary state system reliability analysis availability and unavailability plays a very major role in system maintenance through corrective maintenance of MSSs. A method for the two-sided estimation of MSS unavailability was proposed based on the binary model, Pouret et.al (1999). Large system analysis using extreme value theory is important in the MSS theory. An asymptotic approach to the MSS reliability evaluation was presented in Kolowrocki (2000). Chacko and Manoharan (2009), Chacko et. al. (2018) considered MSSs reliability problems like ageing properties with semi-Markov modeling.

MSS reliability assessment are based on three different approaches Aven (1993): the structure function approach - where Boolean models are extended for the multi-valued case, the stochastic process (mainly Markov) approach, and Monte-Carlo simulation. The structure function approach is also extremely time consuming and difficult to deal with. The stochastic process method can be applied only to relatively small MSS, because the number of system states increases drastically with the increase in number of system components. A Monte-Carlo simulation model may be a fairly true representation of the real world, but the main disadvantage of the simulation technique is the time and expense involved in the development and execution of the model Aven (1993). This is an especially important drawback when the optimization problems are solved. In spite of these limitations, the above mentioned methods are often used by practitioners, for example in the field of power systems reliability analysis Pouret et.al. (1999).

MSSs reliability analysis is more complex in reality. In real-world problems of MSS reliability analysis, the great number of system states that need to be evaluated makes it difficult to use traditional techniques in various optimization problems. The universal generating function (UGF) technique is fast enough to be used in these problems in discrete state MSSs, Ushakov (1986) and Ushakov (1988). In addition, this technique allows practitioners to find the entire MSS performance distribution based on the performance distributions of its components. An engineer can find it by using the same procedures for MSS with different physical nature of performance. In the following sections the application of the UGF to MSS reliability analysis is considered especially for continuum state systems. The discretization of continuous systems make variations in reliability analysis. The results of measure theory and probability theory will become applicable while using continuous performance variables. So it is necessary to study continuous MSSs and introduce analysis tools.

Section II provided the UGF for continuous MSSs. Performance measure evaluation is given in section III. Numerical example is given in section IV. Conclusions are given in final section.

II. Performance Measures of Continuous MSSs

Consider a system consisting of n units. We suppose that any system unit i can have continuous states: from complete failure up to perfect functioning. The entire MSS system has continuous states as determined by the states of its units. Denote a MSS state at instance t as Y(t) $\in [0, b]$, where Y(t)=0 corresponds to the worst state and Y(t)=b corresponds to the best state. The performance level G_y is associated with each state $y \in [0, b]$ and $G_y \geq G_s$ if y>s. The MSS behavior is characterized by its evolution in the space of states. To characterize numerically this evolution process, one has to determine the MSS reliability indices. These indices can be considered as extensions of the corresponding reliability indices for a binary-state system.

The Continuum MSS reliability measures were systematically studied Brunelle and Kapur (1999). In this paper, we consider three measures which are most commonly used by engineers, namely MSS availability, MSS expected performance, and MSS expected unsupplied demand (lost throughput).

MSS availability A(t) is the probability that the MSS will be in the states with performance level greater than or equal to W at a specified moment t>0, where the MSS initial state at the instance t=0 is the best state K or some other predetermined state M (G(y)>W). For large t the initial state has practically no influence on the availability. Therefore, the index A is usually used for the steady state case and is called the stationary availability coefficient, or simply, the MSS availability. MSS availability is the function of required demand W. It may be defined as

$$A(W) = \int_W^{\infty} f(y)dy \tag{1}$$

Where f(y) is the probability density function of MSS performance state y. The resulting integral is taken only for the states where MSS performance is greater than or equal to the specified demand W.

In practice, the system operation period T is often partitioned into M intervals, T_m (1≤m≤M) and each T_m has its own demand level W_m. The following generalization of the availability index as in Levitin et.al. (1998) is used in these cases:

$$E_A = \sum_{m=1}^{M} A(W_m) \cdot q_m, \tag{2}$$

where

$$q_m = T_m / \sum_{m=1}^{M} T_m \tag{3}$$

is the steady state probability of demand level m.

For example, in power system reliability analysis, the index (1-E_A) is often used and treated as loss of load probability, see Billinton and Alen (1996). This measure is commonly used in power system reliability analysis. The MSS performance in this case is interpreted as power system generating capacity.

The value of MSS expected performance could be determined as

$$EG = \int_o^b G(y) f(y) dy. \tag{4}$$

One can note that expected MSS performance does not depend on demand W. EG defines the average productivity (capacity) or processing speed of the system.

When penalty expenses are proportional to the unsupplied demand, the expected unsupplied demand EU may be used as a measure of system output performance. This index may be presented by the following expression:

$$EU = \sum_{m=1}^{M} \int_0^b \max(W_m - G(y) \ \ , 0)\, f(y) dy. q_m, \tag{5}$$

Examples of the EU measure are the unsupplied power in power distribution systems and expected output tardiness in information processing systems. In this case EU may be interpreted as expected electric power unsupplied to consumers.

In the following section we consider MSS reliability assessment based on MSS reliability indices based on the UGF technique.

III. Universal Generating Function of Continuous MSSs

The UGF was introduced in Ushakov (1986) and principles of its application were formulated in Ushakov (1987) and Ushakov (1988). The most systematical description of mathematical aspects of the method can be found in Ushakov (2000), where the method is referred to as generalized generating sequences approach. A brief overview of the method with respect to its applications for MSS reliability assessment can be seen in Levitin et.al (1998). Chacko and Manoharan (2011) discussed application of UGF in finding joint importance measures of MSSs. The method was first applied to the real power system reliability assessment and optimization in Lisnianski et.al (1994,1996)

For MSS which continuous states, there can be different levels of output performance at each time t: G(t)∈**G**={G} and the system output performance distribution (OPD) can be defined by two sets **G** and f(g(t)) .

The u-function of a continuous random variable Y is defined as

$$u(z) = \int_a^b z^y f(y) dy, \tag{6}$$

where the variable G lies between a and b and f(g) is the is the probability density function of G. u-function $u_j(z)$ can be introduced to represent the performance distribution of element j by relating the probabilities of each state y_j, $0 \le y \le b_j$, to the corresponding performance G_{y_j} of the element in that state:

$$u_j(z) = \int_0^{b_j} z^{y_j} f(y_j) dy_j.$$

To obtain the u-function of a subsystem containing two elements, composition operators are introduced. All the composition operators take the form

$$u_i(z) * u_j(z) = \int_0^{b_i} z^{y_i} f_i(y_i) dy_i * \int_0^{b_j} z^{y_j} f_j(y_j) dy_j = \int_0^{b_i}\int_0^{b_j} z^{\omega(y_i, y_j)} f_i(y_i) f_j(y_j) dy_i dy_j$$

The definition of the function $\omega(\cdot)$ strictly depends on the physical nature of the system performance measure and on the nature of the interaction of the system elements, for example for a series system, $\omega(.) = \min(.,.)$, and for a parallel system, $\omega(.) = sum(.,.)$ *or max(.,.).* Because, the total performance of a pair of elements connected in parallel is equal to the sum of the performance rates (e.g. productivity and capacity) of the individual and when several elements are connected in series, the element with the lowest performance becomes the bottleneck of the subsystem: in other words, the performance of the subsystem is equal to the minimum of the performances of the individual.

Consecutively applying the operators to all elements and replacing pairs of macro-elements by equivalent elements one can obtain the u-function representing the performance distribution of the entire MSS. Obtain the u-functions of all of the system elements. If the system contains a pair of elements connected in parallel or in series, replace this pair with an equivalent macro-element with u-function obtained by 'sum' or 'min' operator for $\omega(\cdot)$. If the system contains more than one element, do it again and again. Then, determine the u-function of the entire series-parallel system as the u-function of the remaining single equivalent macro-element. The system probability and performance density functions f(.), g are represented by the coefficients and exponents of this u-function.

In order to use the UGF in evaluation in various reliability measures, we consider the following approach. Let g_{jy_j} be the output performance of multistate system when element *j* is in state y_j while the rest of the elements evolve stochastically among their corresponding states with performance distributions. $f_i(y_i)$, $0 \le y_i \le b_i, 1 \le i \le n$. Assume that the element *j* is in one of its states y_j with performance not greater than α. We denote by $y_{j\alpha}$ the state in the ordered set of states of element *j* whose performance $g_{jy_{j\alpha}}$ is equal or immediately below α, i. e., $g_{jy_{j\alpha}} \le \alpha \le g_{jy_{j\alpha+}}$. The conditional probability of the element *j* being in a generic state *k* characterized by a performance $g_{jy_{j\alpha}}$ not greater than a pre-specified level threshold α is

$$f_j\left(Y_j = y_j \middle| G_j \le g_{jy_{j\alpha}}\right) = \frac{f_j(Y_j = y_j)}{F(g_{jy_{j\alpha}})}$$

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Similarly, the conditional probability of the element *j* being in a generic state *k* characterized by a performance $g_{jy_{j\alpha}}$ greater than a pre-specified level threshold α is

$$f_j\left(Y_j = y_j \middle| G_j > g_{jy_{j\alpha}}\right) = \frac{f_j(Y_j = y_j)}{1 - F(g_{jy_{j\alpha}})}$$

In this model we get $OPM^{\le\alpha}{}_j$:

$$OPM^{\le\alpha}{}_j = \int_0^{g_{jy_{j\alpha}}} Y_j\ f_j\left(Y_j = y_j \middle| G_j \le g_{jy_{j\alpha}}\right)\ dy_j = \int_0^{g_{jy_{j\alpha}}} Y_j \frac{f_j(Y_j = y_j)}{F(g_{jy_{j\alpha}})} dy_j$$

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Similarly, we define as $OPM^{>\alpha}{}_j$:

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$$OPM^{>\alpha}{}_j = \int_{g_{jy_{j\alpha}}}^{b_j} Y_j\ f_j\left(Y_j = y_j \middle| G_j > g_{jy_{j\alpha}}\right)\ dy_j = \int_{g_{jy_{j\alpha}}}^{b_j} Y_j \frac{f_j(Y_j = y_j)}{1 - F(g_{jy_{j\alpha}})} dy_j$$

In order to obtain the state space restricted measures, one has to modify the UGF of elements as follows,

$$U^{\le\alpha}{}_j = \int_0^{g_{jy_{j\alpha}}} z^{y_j}\ f_j\left(Y_j = y_j \middle| G_j \le g_{jy_{j\alpha}}\right)\ dy_j = \int_0^{g_{jy_{j\alpha}}} z^{y_j} \frac{f_j(Y_j = y_j)}{F(g_{jy_{j\alpha}})} dy_j$$

$$U^{>\alpha}{}_j = \int_{g_{jy_{j\alpha}}}^{b_j} z^{y_j}\, f_j\left(Y_j = y_j \middle| G_j > g_{jy_{j\alpha}}\right)\, dy_j = \int_{g_{jy_{j\alpha}}}^{b_j} z^{y_j} \frac{f_j(Y_j = y_j)}{1 - F(g_{jy_{j\alpha}})} dy_j$$

$$U_{j,k}{}^{\le\alpha,\le\beta} = \int_0^{g_{jy_{j\alpha}}} z^{y_j} \frac{f_j(Y_j = y_j)}{F(g_{jy_{j\alpha}})} dy_j \int_0^{g_{ky_{k\beta}}} z^{y_k} \frac{f_k(Y_k = y_k)}{F(g_{ky_{k\beta}})} dy_k$$

$$U_{j,k}{}^{>\alpha,>\beta} = \int_{g_{jy_{j\alpha}}}^{b_j} z^{y_j} \frac{f_j(Y_j = y_j)}{1 - F(g_{jy_{j\alpha}})} dy_j \int_{g_{ky_{k\beta}}}^{b_k} z^{y_k} \frac{f_k(Y_k = y_k)}{1 - F(g_{ky_{k\beta}})} dy_k$$

$$U_{j,k}{}^{\le\alpha,>\beta} = \int_0^{g_{jy_{j\alpha}}} z^{y_j} \frac{f_j(Y_j = y_j)}{F(g_{jy_{j\alpha}})} dy_j \int_{g_{ky_{k\beta}}}^{b_k} z^{y_k} \frac{f_k(Y_k = y_k)}{1 - F(g_{ky_{k\beta}})} dy_k$$

$$U_{j,k}{}^{>\alpha,\le\beta} = \int_{g_{jy_{j\alpha}}}^{b_j} z^{y_j} \frac{f_j(Y_j = y_j)}{1 - F(g_{jy_{j\alpha}})} dy_j \int_0^{g_{ky_{k\beta}}} z^{y_k} \frac{f_k(Y_k = y_k)}{F(g_{ky_{k\beta}})} dy_k$$

when evaluating UGF of

$$OPM^{\le\alpha}{}_i, OPM^{>\alpha}{}_i, OPM^{\le\beta}{}_j, OPM^{>\beta}{}_j, OPM^{\le\beta,\le\alpha}{}_{ij}, OPM^{>\beta,>\alpha}{}_{ij}, OPM_{ij}^{\le\alpha,>\beta} and\ OPM_{ij}^{>\alpha,\le\beta}.$$

Having MSS OPD, one can obtain the system availability for the arbitrary t and W using the following operator δ_A:

$$A(t,W) = \delta_A(U_{MSS}(t,z),W) = \delta_A(\int_0^b Z^y f(y(t)dy,W) = \int_0^b f(y(t))\alpha(y(t)-W)dy \quad (7)$$

where

$$\alpha(x) = \begin{cases} 1, x \ge 0, \\ 0, x < 0. \end{cases} \quad (8)$$

The expected system output performance value during the operating time can be obtained for given $U_{MSS}(z)$ using the following δ_G operator:

$$E_G(t) = \delta_G(U_{MSS}(t,z)) = \delta_G(\int_0^b Z^y f(y(t)dy) = \int_0^b y(t) f(y(t)dy \quad (9)$$

In order to obtain the expected unsupplied demand EU for the given $U_{MSS}(z)$ and constant demand W according to (4), the following δ_U operator should be used:

$$E\ U(t) = \delta_U(U_{MSS}(t,z)) = \delta_U(\int_0^b Z^y f(y(t)dy,W) = \int_0^b \max(0, W - y(t)) f(y(t)dy$$

$$EU(\mathbf{W}) = \sum_{m=1}^{M} q_m \delta_U(U_{MSS}(z),W_m), \quad (10)$$

where

$$EU(W_m) = \delta_U(U_{MSS}(z),W_m) = \delta_U(\int_0^b Z^y f(y(t)dy,W_m) = \int_0^b \max(0,W_m - y(t)) f(y(t))dy$$

Consider, for example, two power system generators with nominal capacity 100 MW as two separate MSS, Billonton (1996). In the first generator some types of failures require the capacity to be reduced to 60 MW and some types lead to the complete generator outage. In the second one some types of failures require the capacity to be reduced to 80 MW, some types lead to capacity reduction to 40 MW and some types lead to the complete generator outage. So, there are three possible relative capacity levels that characterize the performance of the first generator:

Universal Generating function (UGF) is found to be a useful toll in determining the system performance for the MSSs. Real world MSS are often very complex and consist of a large number of elements connected in different ways. To obtain the MSS OPD and the corresponding u-function, for the continuum state MSSs, we must develop some rules to determine the system u-function based on the individual u-function of its elements.

In order to obtain the u-function of a subsystem (component) containing a number of elements, composition operators are introduced. These operators determine the subsystem u-function expressed as integral for a group of elements using simple algebraic operations over individual u-functions of elements. All the composition operators for two different elements take the form

$$\Omega_\omega(u_1(t,z),u_2(t,z))=\Omega_\omega[\int_0^{b_1} Z^{y_1(t)} f_1(y_1(t))dy_1, \int_0^{b_2} Z^{y_2(t)} f_2(y_2(t))dy_2]=\int_0^{b_1}\int_0^{b_2} Z^{\omega(y_1(t),y_2(t))} f_1(y_1(t)) f_2(y_2(t))du_1du_2, \quad (11)$$

where $u_1(t,z)$, $u_2(t,z)$ are individual U-function of elements and ω(·) is a function that is defined according to the physical nature of the MSS performance and the interactions between MSS elements. The function ω(·) in composition operators expresses the entire performance of a subsystem consisting of different elements in terms of the individual performance of the elements. The definition of the function ω(·) strictly depends on the type of connection between the elements in the reliability diagram sense, i.e. on the topology of the subsystem structure. It also depends on the physical nature of system performance measure.

For example in MSS, where performance measure is defined as capacity or productivity (MSSc), the total capacity of a pair of elements connected in parallel is equal to the sum of the capacities of elements. Therefore, the function ω(·) in composition operator takes the form:

$$\omega(g_1,g_2)=g_1+g_2. \quad (12)$$

For a pair of elements connected in series the element with the least capacity becomes the bottleneck of the system. In this case, the function ω(·) takes the form:

$$\omega(g_1,g_2)=\min(g_1,g_2). \quad (13)$$

In MSS where the performances of elements are characterized by their processing speed (MSSs) and parallel elements cannot share their work, the task is assumed to be completed by the group of parallel elements when it is completed by at least one of elements. The entire group processing speed is defined by the maximum element processing speed:

$$\omega(g_1,g_2)=\max(g_1,g_2). \quad (14)$$

If a system contains two elements connected in series, the total processing time is equal to the sum of processing times t_1 and t_2 of individual elements: $T=t_1+t_2=g^{-1}_1+g^{-1}_2$.

Therefore, the total processing speed of the system can be obtained as $T^{-1}=g_1g_2/(g_1+g_2)$ and the ω(·) function for a pair of elements is defined as follows:

$$\omega(g_1,g_2)=g_1g_2/(g_1+g_2). \quad (15)$$

Ω operators were determined in Levitin et.al (1998), Lisnianski et.al (2000) for several important types of series-parallel systems MSS. Some additional composition operators were also derived for bridge structures Levitin and Lianianski (1998).

Applying the Ω operators in sequence, one can obtain the u-function representing the system performance distribution for an arbitrary number of elements connected in series, in parallel, or forming bridge structure.

If Y_1 follows Exp(θ) and Y_2 follows Exp(μ), then

$$\Omega_\omega(u_1(t,z),u_2(t,z))=\int_0^{\infty}\int_0^{\infty} Z^{y_1(t)+y_2(t)}\theta\mu e^{-\theta y_1(t)-\mu y_2(t)}dy_1dy_2$$

for parallel structure

$$\Omega_\omega(u_1(t,z),u_2(t,z)) = \int_0^\infty\int_0^\infty Z^{\min(y_1(t),y_2(t))}\theta\mu e^{-\theta y_1(t)-\mu y_2(t)}dy_1dy_2$$

for series structure

$$\Omega_\omega(u_1(t,z),u_2(t,z)) = \int_0^\infty\int_0^\infty e^{[y_1(t)+y_2(t)]\log Z}\theta\mu e^{-\theta y_1(t)-\mu y_2(t)}dy_1dy_2$$

$$\Omega_\omega(u_1(t,z),u_2(t,z)) = \int_0^\infty\int_0^\infty \theta\mu e^{-(\theta-\log Z)y_1(t)-(\mu-\log Z)y_2(t)}dy_1dy_2 = \frac{\theta\mu}{(\theta-\log Z)(\mu-\log Z)}$$

Putting z=1, we get output performance for parallel structure. Similarly for series system.

$$A_{Series}(u_1(t,z),u_2(t,z)) = I_{(z=1,g_1(t)\geq W\ ,g_2(t)\geq W)}\int_0^\infty\int_{0_2}^\infty \theta\mu e^{-(\theta)y_1(t)-(\mu)y_2(t)}dy_1dy_2 = \frac{\theta\mu e^{-\theta y_w\ -\mu y_w}}{(\theta)(\mu)} = e^{-\theta y_w-\mu y_w}$$

$$A_{Parallel}(u_1(t,z),u_2(t,z),W) = 1-I_{(z=1,g_1(t)+g_2(t)\leq W)}\int_0^\infty\int_0^\infty \theta\mu e^{-(\theta)y_1(t)-(\mu)(y_2(t))}dy_2dy_1$$

$$A_{Parallel}(u_1(t,z),u_2(t,z),W) = 1-\int_0^\infty\int_0^{W-u_1(t)} \theta\mu e^{-(\theta)u_1(t)-(\mu)(u_2(t))}du_2du_1 = 1-\int_0^\infty \theta e^{-\theta u_1(t)}(1-e^{-\mu(w-u_2(t))})du_1$$

$$A_{Parallel}(u_1(t,z),u_2(t,z),W) = 1-\int_0^\infty \theta e^{-\theta u_1(t)}(1-e^{-\mu(w-u_1(t))})du_1 = \frac{\theta}{\theta-\mu}e^{-\mu w}$$

$$A_{Parallel}(u_1(t,z),u_2(t,z),W) = 1-I_{(z=1,y_1(t)\leq w,y_2(t)\leq W)}\int_0^\infty\int_0^\infty \theta\mu e^{-(\theta)y_1(t)-(\mu)(y_2(t))}dy_2dy_1$$

$$A_{Parallel}(u_1(t,z),u_2(t,z),W) = 1-\int_0^W\int_0^W \theta\mu e^{-(\theta)y_1(t)-(\mu)(y_2(t))}dy_2dy_1 = 1-(1-e^{-\theta W})(1-e^{-\mu W})$$

$$F(w) = 1-\frac{\theta}{\theta-\mu}e^{-\mu w}$$

$$\Omega_\omega(u_1(t,z),u_2(t,z)) = \int_0^\infty\int_0^\infty Z^{\min(y_1(t),y_2(t))}\theta\mu e^{-\theta y_1(t)-\mu y_2(t)}dy_1dy_1$$

$$A_{Series}(u_1(t,z),u_2(t,z)) = I_{(z=1,y_1(t)0\geq W,y_2(t)\geq W)}\int_0^\infty\int_0^\infty \theta\mu e^{-(\theta)y_1(t)-(\mu)y_2(t)}du_1du = \frac{\theta\mu e^{-\theta w-\mu w}}{(\theta)(\mu)} = e^{-(\theta+\mu)w}$$

IV. Numerical Example

Time to failure of two components an a system is given in table 1. The availability is estimated if the components are connected in series and in parallel. Both of the data follows exponential distribution, since the failures are due to shocks occurred during operation. The parameters are estimated and availability formula is obtained.

The data is found to be follows Exponential distribution with mean 19.22 and 27.54 respectively.

If the components are connected in series, the availability would be, at w,

$$A_{Series}(u_1(t,z),u_2(t,z)) = e^{-46.76w}$$

If the components are connected in parallel, the availability would be, at w,

$$A_{Parallel}(u_1(t,z),u_2(t,z),W) = 1-\int_0^W\int_0^W \theta\mu e^{-(\theta)y_1(t)-(\mu)(y_2(t))}dy_2dy_1 = 1-(1-e^{-19.22W})(1-e^{-27.54W})$$

The availability for various w values van be easily obtained.

Table 1: Time to failure of two components (1 and 2)

Time of failure (Component 1)	Time of failure (Component 2)
4.6	15.0
5.6	7.2
6.6	8.5
7.6	9.8
8.6	11.2
9.7	32.0
10.8	14.0
11.9	15.5
13.1	18.0
14.3	18.5
17.0	17.0
16.7	21.8
18.0	53.0
19.4	72.0
20.8	27.0
22.2	30.0
22.0	22.0
25.2	32.7
26.8	35.0
28.4	36.9
31.0	31.0
31.9	41.4
33.7	43.8
36.0	36.0
39.0	39.0

V. Conclusions

The Universal Generating Function for continuous performance distributions are introduced. Discretization of continuous system becomes sometimes more unrealistic inferences. Method for analyzing continuous MSSs is desired. In this paper, we have made an attempt to deal with continuous MSSs, which will guide to obtain performance measures of complex MSSs. More analysis has to be explored in future.

References

[1] Aven T. (1990) Availability evaluation of flow networks with varying throughput-demand and deferred repairs. *IEEE Transactions on Reliability*; **38**: 499-505.
[2] Billinton R, Allan R. *Reliability Evaluation of Power System*s, Plenum Press, 1996
[3] Chacko V. M. and Manoharan M (2008) Joint Importance measures for the Multistate systems,*Advances in Safety and Performance of Complex Systems*, Advances in Safety and

Performance of Complex Systems, A. K. Verma, P. K. Kapur, S. G. Gadge (Eds), Macmillan, India, 308-312, 2008

[4] Chacko V M and Manoharan M (2009) Ageing properties of first passage time distribution of multistate reliability system, *ProbStat Forum, An e-journal in Probability, Statistics and Optimizati*on, Vol.2, 22-30, Jan-2009

[5] Chacko V M and Manoharan M (2011) Joint Importance measures for multistate reliability system, Opsearch (Available online) Vol.48, No.3, 257-278, 2011

[6] Chacko V M, Deepa Paul and Rajitha C (2018) Ageing properties of First passage time of Semi-Markov system, Birth-Death models and Queuing Models, Advances in Mathematical Sciences, Ed. Dr.Ratnakar D B, Dr.Sr.Alphonsa Mathew, Appeared in International Journal of Mathematical Sciences, 2018

[7] Levitin G, Lisnianski A, Ben-Haim H, Elmakis D. (1998) Redundancy optimization for series-parallel multi-state systems. *IEEE Transactions on Reliability*; **47**:(2):165-172.

[8] Lisnianski A, Levitin G, Ben-Haim H. (2000) Structure optimization of multi-state system with time redundancy. *Reliability Engineering and System Safety*; 67: 103-112.

[9] Gnedenko B, Ushakov I.(1995) *Probabilistic Reliability Engineering*. John Wiley & Sons, Inc, NY/Chichester/Brisbane.

[10] Murchland J. (1975) Fundamental concepts and relations for reliability analysis of Multistate systems, Reliability and Fault Tree Analysis, in *Theoretical and Applied Aspects of System Reliability*; SIAM: 581-618.

[11] El-Neveihi E, Proschan F, Setharaman J. (1978) Multistate coherent systems, *J. Applied Probability*; **15**: 675-688.

[12] Barlow R, Wu A. (1978) Coherent systems with multistate components. *Math. Operation Research*; **3**: 275-281.

[13] Ross S. (1979) Multivalued state component systems. *Annals of Probability* ; **7**: 379-383.

[14] Griffith W. (1980) Multistate reliability models. *J. Applied Probability*; **17**: 735-744.

[15] Butler D. (1979) A complete importance ranking for components of binary coherent systems with extensions to multistate systems. *Naval Research Logistics Quarterly*; **26**: 556-578.

[16] Block H, Savits T.(1982) Decomposition of Multistate monotone system. *J. Applied Probability*; **19**: 391-402.

[17] Natvig B, Streller A. (1984) The steady-state behavior of multistate monotone systems. *J. Applied Probabilit*; **21**: 826-835.

[18] Wood A. (1985) Multistate Block Diagrams and Fault Trees. *IEEE Transactions on Reliability,* **34**: 236-240.

[19] Garriba S, Mussio P, Naldi F. (1980) Multivalued logic in the representation of engineering systems, *in Synthesis and analysis methods for safety and reliability studies,* Apostolakis G, Garriba S, Volta G. eds., Plenum Press; 183-197.

[20] Yau M, Apostolakis G, Guarro S. (1998) The use of prime imlicants in dependability analysis of software controlled systems. *Reliability Engineering and System Safety*; **62**: 23-32.

[21] Pouret O, Collet J, Bon J-L. (1999) Evaluation of the unavailability of a multistate-component system using a binary Model. *Reliability Engineering & System Safety*; **66**: 13-17.

[22] Kolowrocki K.(2000), An asymptotic approach to multistate systems reliability evaluation, in Recent advances in reliability theory, methodology, practice and inference, Limnios N, Nikulin M, eds., Birkhauser: 163-180.

[23] Aven T. (1993) On performance measures for multistate monotone systems, *Reliability Engineering and System Safety*: **41**: 259-266.

[24] Painton L, Campbell J. (1995) Genetic algorithm in optimization of system reliability, *IEEE Trans. Reliability*: **44**: 172-178.

[25] Coit D, Smith A (1996) . Reliability optimization of series-parallel systems using genetic algorithm, IEEE Trans. Reliability; **45**: 254-266.

[26] Yang J, Hwang M, Sung T, Jin Y. (1999) Application of genetic algorithm for reliability allocation in nuclear power plant, *Reliability Engineering & System Safety*; **65**: 229-238.

[27] Gen M, Kim J. (1999) GA-based reliability design: state-of-the-art survey, *Computers ind. Engn*; **37**: 151-155.

[28] Gen M, Cheng R. (1997) *Genetic Algorithms and engineering design*, John Wiley & Sons, New York.

[29] Kumar A, Pathak R, Gupta Y, (1995) Genetic algorithm-based reliability optimization for computer network expansion, *IEEE Trans. Reliability*; **44**: 63-72.

[30] Bartlett L, Andrews J. (1999) Efficient basic event ordering schemes for fault tree analysis, *Quality and reliability engineering International*; **15**: 95-101.

[31] Shao S, Murotsu Y. (1999) Approach to failure mode analysis of large structures, Probabilistic engineering Mechanics;**14**: 169-177.

[32] Brunelle R, Kapur K. (1999) Review and classification of reliability measures for multistate and continuum models. *IIE Transactions*; **31**: 1171-1180.

[33] Ushakov I. (1986) A universal generating function. *Soviet J. Comput. Systems Sci.* 1986; **24**: 37-49.

[34] Ushakov I.(1987) Optimal standby problem and a universal generating function. *Soviet J. Comput. Systems Sci.* 7; **25**: 61-73.

[35] Ushakov I. (1988) Reliability analysis of multi-state systems by means of a modified generating function. *J. Inform. Process. Cybernet.*; **34**: 24-29.

[36] Ushakov I. (2000)The method of generalized generating sequences, *European Journal of Operating Research*; **125**: 2: 316-323.

[37] Abouammoh, A. M. and Al-kadi (1991). On measures of importance for components in multi-state coherent system, Microel. Reliab., 31, 109-122.

[38] Abouammoh, A. M. and Al-kadi (1995). Multi-state coherent system of order k, Microel. Reliab., 35, 1415-1421.

[39] Barlow, R. E. and Proschan, F. . Statistical Theory of Reliability and Life Testing, Holt, Rinehart and Winston, New York, 1975

[40] Belzunce, F., Ortega, E. M. and Ruizon, J. M. (2002). Ageing properties of first- passage times of increasing Markov processes, Adv. Appl. Prob., 34, 241-259.

[41] Brown, M. and Chaganty, N. R. (1983). On the first passage time distribution for a class of Markov chains, Ann. Prob., 11, 1000-1008.

[42] Bueno, V. C. (1989). On the importance of components for multi-state monotone systems, Stat. Prob. Lett., 7, 51-59.

[43] Deshpande, J. V., Kochar, S. C. and Singh, H. (1986). Aspects of positive aging, J. Appl. Prob., 23, 748-758.

[44] El-Neweihi, E., Proschan, F., and Sethuraman, J. (1978). Multi-state coherent systems, J. Appl. Prob., 15, 675-688.

[45] Hjort, N. B., Natvig, B. and Funnemark, E. (1985). The association in time of a Markov process with applications to multi-state reliability theory, J. Appl. Prob., 22 473-479.

[46] Lisnianski, A. and Levitin, G. Multi-state System Reliability-Assessment, Optimization and Applications, World Scientific, Singapore (2003).

[47] Meng, F. C. (1993). Component relevancy and characterization results in multi-state systems, IEEE Trans. Reliab., 42, 478-483.

[48] Wu, S. (2005). Joint importance of multi-state systems, Comp. Indust. Eng., 49, 63-75.

[49] Zio, E. and Podofillini, L. (2003). Importance measures of multi-state components in multi-state systems, In. J. Reliab. Qual. Saft. Eng., 10, 289-310.