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CHAPTER 7

BURN-IN PROCESS USING BFR DISTRIBUTIONS

7.1 Introduction

Burn-in testing (BIT) is a widely accepted method for many years to detect and eliminate early failures. BIT is mandatory in high-reliability storage contracts such as military and aerospace applications, and is also essential for automotive, medical, long-distance telecommunications, and other electronic materials, packages and systems. BIT is usually performed at the component level because the cost involved in inspecting and replacing parts is small.

Testing plays an important role in controlling and ensuring the required quality and reliability of built-in integrated circuits (IC). The IC fabrication process involves several tests at different stages: pre-burn-in, burn-in and final test (see Kececioglu and Sun (1997)). The four main types of burn-in tests used in the industry are static, dynamic, monitored and test-in burn-in (TIBI) (Ooi et al. (2007)). In static burn-in (also known as traditional burn-in) equipment under test (EUT) is subjected to high temperatures. Dynamic burn-in is similar to static one which involves exercising EUTs by applying test vectors or stimulus sets to toggle the device's internal nodes. Static or dynamic burn-in types provide no monitoring of EUT responses. Consequently faulty ICs are not detected until a subsequent final test stage.

The burn-in procedure stops when we get pre-determined reliability. In Mi (1994a) it was shown that the optimal burn-in time, say b^*, for maximizing the mean residual life function $\mu(b) = E(X - b|X > b)$ satisfies $b^* \leq t_1$, where t_1 is the first change point, if F is BFR. Since burn-in is usually expensive, an important issue is deciding how long the procedure should continue. The time to stop the burn-in process to optimize a given criterion is known as optimal burn-in time (see, Jensen and Petersen (1982)).

For burn-in to be effective, it must have a high failure rate early in life. Items that survive the burn-in have burn-in effect that eliminates the part of the lifetime with a high initial risk of failure. A class of life time distributions with bathtub-shaped failure rates has this property. Some other mechanical and electronic lifetimes can also be analyzed by BFR distributions.

An engineer thinking about burn-in use needs to answer a number of questions related to the purpose of the burn-in test, the type of lifetime supply, the availability of data, and the logistics of running the procedure. Lawrence (1966) and Chandrasekaran (1977) investigated the burn-in problems. Park (1985) learned about the mean residual life of the product. Plaser and Fied (1977), and Nguyen and Murthy (1982) studied the Economic design of burn-in procedures. Li and Cheng (2010) studied the best designs of accelerated life tests for lives that are distributed as exponential under advanced censoring.

In this chapter we discussed optimal burn-in process and expression of long run average cost function per unit time for obtaining optimal burn-in time and optimal age using WL and GXE distribution.

7.2 Optimal Burn-in

Traditionally, burn-in has been used to increase the mean residual life of items that survive the burn period. There are situations where increasing the mean residual life expectancy is not an appropriate criterion. For example, when considering an item going on a space mission, the goal is to minimize the chances of failing on a mission over a period of time. The optimal burn-in time may be different from the optimal burn-in time with the maximum mean residual life. Another goal of burn-in is to achieve a certain degree of reliability. Costs often need to be considered in the goal of a burn-in procedure. The cost of a component failure is higher for a satellite than for a vehicle battery (see, Myung and Young (2002)).

In this section some basic criteria for determining the optimal burn-in time for a lifetime is discussed. In Section 7.3.1 performance based criteria is discussed.

The maintenance policy in burn-in is considered in Section 7.3.2.

7.2.1 Performance-Based Criteria

Consider the performance-based criteria by maximizing the average remaining life, in which there is no more general understanding of the cost structure. We now list several criteria for determining burn-in (see, Block and Savits (1997)).

C1: Let T be a fixed mission time and let $\bar{F}$ be the survival function of the lifetime random variable. Find b which maximizes $\bar{F}(b+T)/\bar{F}(b)$, that is, find b such that, given survival to time b, the probability of completing the mission is as large as possible.

C2: Let X be a lifetime random variable. Find the burn-in time b which maximizes $E(X-b|X>b)$, that is, find the burn-in time which gives the largest mean residual life.

C3: Let $\{N_b(t),\ t \geq 0\}$ be a renewal process of lifetimes which are burned in for b units of time (i.e., if F is the original lifetime distribution and the inter-arrival distribution has survival function $\bar{F}_b(t) = \bar{F}(b+Tt)/\bar{F}(b)$. For fixed mission time T, find b which minimizes $E\left[N_b(t)\right]$, which is the mean number of burn-in components which fail during the mission time T.

C4: For a fixed α, $0 < \alpha < 1$, find the burn-in time b which maximizes $T = q_\alpha(b)$, where $q_\alpha(b) = F_b^{-1}(\alpha) = \inf\{x \geq 0 : \bar{F}_b(x) \leq 1-\alpha\}$ is α-percentile residual life (see Joe and Proschan(1984)), i.e., find the burn-in time which gives the maximal warranty period T for which at most $\alpha\%$ of items will fail.

Criteria C1, C2, and C4 deal with only one component. Criterion C3 deal with replacement components with other similar components when they fail. Mi (1994b) achieved similar results to the C1 and C3 criteria. Launer (1993) showed that optimal burn-in time occurs before t_1.

7.2.2 Burn-in and Maintenance Policy

The most common replacement policy would be age replacement policy, in which component is replaced at time T or at the time of failure which occurs first. Once a cost structure has been established, to model the total cost related to the maintenance policy adopted, an optimal T is determined (denoted by T^* and is called optimal maintenance policy) such that costs will be minimized. Assuming that the failure rate increases, Barlow and Proschan (1975) have shown that an optimal age replacement policy exists, but it may be infinite. The optimal maintenance policy, however, depends on the distribution of the component used in the operation.

Mi (1994a) consider the following procedure. Consider burn-in a new component. If the component fails before burn-in time b, repair it, and then re-burn the component. If this element survives the time b, it can be used for operation. Cha (2000) adopted a block replacement policy with fewer failures.

For new component having burn-in time b, if failure occurs before the b, then minimal repair will carry out with cost $c_s > 0$, and the burn-in procedure will continue for the repaired component. Let c_o be the cost to burn which is calculated in proportion to the total burn-in time, the total expected cost incurred by burn-in is the sum of the cost for burn-in $c_o b$, and the expected cost of minimal repairs $c_s \int_0^b r(t)\ dt$.

$$i.e.,\ C_1(b) = c_o b + c_s \int_0^b r(t)\ dt \tag{7.2.1}$$

where $\int_0^b r(t)\ dt$ is the expected no.of minimal repairs during the burn-in period.

Let c_f indicates the cost incurred by the replacement at age T and c_r be the cost incurred by failure replacement before T^*, $0 < c_r < c_f$. Then the total expected replacement cost is the sum of the expected cost incurred by replacement at age T and the expected cost incurred by failure replacement before T^*,

$$C_2(T) = c_f F_b(T) + c_r \bar{F}_b(T) \tag{7.2.2}$$

where $\bar{F}_b(T)$ is the conditional survival function $\frac{\bar{F}(b+T)}{\bar{F}(b)}$, then $F_b(T) = 1 - \bar{F}_b(T)$.

The mean residual life function for a general repairable product is $\mu(b) = \frac{\int_b^\infty \bar{F}(t)\ dt}{\bar{F}(b)}$. The total expected cycle length is the sum of the expected length of a replacement for non-failed item and the expected length of failure cycle;

$$T\bar{F}_b(t) + \int_0^T t f_b(t)\ dt = \int_0^T \bar{F}_b(t)\ dt. \tag{7.2.3}$$

Hence, from (7.2.1), (7.2.2) and (7.2.3) the long-run average cost per unit time $C(b,T)$ is

$$\begin{aligned} C(b,T) &= \frac{c_o + c_s \int_0^b r(t)\ dt + c_f F_b(T) + c_r \bar{F}_b(T)}{\int_0^T \bar{F}_b(t)\ dt} \\ &= \frac{\left(c_o + c_s \int_0^b r(t)\ dt\right) \bar{F}(b) + c_f \left(\bar{F}(b) - \bar{F}(b+T)\right) + c_r \bar{F}(b+T)}{\int_0^T \bar{F}(b+t)\ dt}. \end{aligned}$$

The optimal burn-in time b^* and the optimal age T^* which satisfy $C(b^*, T^*) = \min_{b\geq 0, T>0} C(b, T)$.

7.3 Optimal Burn-in Procedure for WL and GXE distributions

WL Distribution:- Let X be a lifetime r.v. following WL distribution with failure rate function

$$r(x) = \alpha\Big(\beta x^{\beta-1}(1+x)e^{x^\beta} + e^{x^\beta}\Big), \quad x > 0,\ \alpha > 0,\ \beta > 0$$

and cdf

$$F(x; \alpha, \beta) = 1 - e^{-\alpha\big((1+x)e^{x^\beta}-1\big)}, \quad x > 0,\ \alpha > 0,\ \beta > 0.$$

The total expected cost incurred by burn-in is

$$\begin{aligned} C_1(b) &= c_o b + c_s \int_0^b \alpha\Big(\beta t^{\beta-1}(1+t)e^{t^\beta} + e^{t^\beta}\Big)\, dt \\ &= c_o b + c_s\{\alpha(1+b)e^{b^\beta}\} \end{aligned} \tag{7.3.1}$$

$$\bar{F}_b(T) = \frac{e^{-\alpha\big((1+(b+T))e^{(b+T)^\beta}-1\big)}}{e^{-\alpha\big((1+b)e^{b^\beta}-1\big)}} \tag{7.3.2}$$

$$F_b(T) = 1 - \left\{\frac{e^{-\alpha\big((1+(b+T))e^{(b+T)^\beta}-1\big)}}{e^{-\alpha\big((1+b)e^{b^\beta}-1\big)}}\right\}. \tag{7.3.3}$$

Substituting (7.3.2) and (7.3.3) in (7.2.2), we get total expected replacement cost as

$$C_2(T) = c_f \left(1 - \left\{ \frac{e^{-\alpha\left((1+(b+T))e^{(b+T)^\beta}-1\right)}}{e^{-\alpha\left((1+b)e^{b^\beta}-1\right)}} \right\} \right) + c_r \left\{ \frac{e^{-\alpha\left((1+(b+T))e^{(b+T)^\beta}-1\right)}}{e^{-\alpha\left((1+b)e^{b^\beta}-1\right)}} \right\}. \tag{7.3.4}$$

The total expected cycle length is

$$\begin{aligned} &T\frac{e^{-\alpha\left((1+(b+t))e^{(b+t)^\beta}-1\right)}}{e^{-\alpha\left((1+b)e^{b^\beta}-1\right)}} + \int_0^T t\alpha \left[\beta(b+t)^{\beta-1}(1+(b+t))e^{(b+t)^\beta}\right] \\ &\times \frac{e^{-\alpha\left((1+(b+t))e^{(b+t)^\beta}-1\right)}}{e^{-\alpha\left((1+b)e^{b^\beta}-1\right)}}\, dt = \int_0^T \bar{F}_b(t)\, dt. \end{aligned} \tag{7.3.5}$$

Hence, from (7.3.1), (7.3.4) and (7.3.5), the long-run average cost per unit time is

$$C(b,T) = \frac{\left(c_o + c_s\{\alpha(1+b)e^{b^\beta}\}\right) e^{-\alpha\left((1+b)e^{b^\beta}-1\right)}}{\int_0^T e^{-\alpha\left((1+(b+t))e^{(b+t)^\beta}-1\right)}\, dt}$$

$$\begin{aligned} &+ \frac{c_f\left(e^{-\alpha\left((1+b)e^{b^\beta}-1\right)} - e^{-\alpha\left((1+(b+T))e^{(b+T)^\beta}-1\right)}\right)}{\int_0^T e^{-\alpha\left((1+(b+t))e^{(b+t)^\beta}-1\right)}\, dt} \\ &+ \frac{c_r e^{-\alpha\left((1+(b+T))e^{(b+T)^\beta}-1\right)}}{\int_0^T e^{-\alpha\left((1+(b+t))e^{(b+t)^\beta}-1\right)}\, dt}. \end{aligned}$$

GXE Distribution:- Let X be a lifetime r.v. following the failure rate function GXE distribution with $x > 0,\ \alpha, \lambda > 0$,

$$r(x) = \frac{\alpha e^{-\lambda(x^2+x)}(\lambda(1+\lambda x^2)(2x+1) - 2\lambda x)\left(1-(1+\lambda x^2)e^{-\lambda(x^2+x)}\right)^{\alpha-1}}{1-(1-(1+\lambda x^2)e^{-\lambda(x^2+x)})^\alpha}$$

and cdf

$$F(x) = \left(1 - (1 + \lambda x^2)e^{-\lambda(x^2+x)}\right)^{\alpha}, \quad x > 0,\ \alpha > 0,\ \lambda > 0.$$

The total expected cost incurred by burn-in is

$$\begin{aligned} C_1(b) &= c_o b \\ &+ c_s \int_0^b \frac{\alpha e^{-\lambda(t^2+t)}(\lambda(1+\lambda t^2)(2t+1) - 2\lambda t)\left(1 - (1+\lambda t^2)e^{-\lambda(t^2+t)}\right)^{\alpha-1}}{1 - (1 - (1+\lambda t^2)e^{-\lambda(t^2+t)})^{\alpha}}\, dt \end{aligned} \tag{7.3.6}$$

$$\bar{F}_b(T) = \frac{1 - \left(1 - (1+\lambda(b+T)^2)e^{-\lambda((b+T)^2+(b+T))}\right)^{\alpha}}{1 - (1 - (1+\lambda b^2)e^{-\lambda(b^2+b)})^{\alpha}} \tag{7.3.7}$$

$$F_b(T) = 1 - \left\{\frac{1 - \left(1 - (1+\lambda(b+T)^2)e^{-\lambda((b+T)^2+(b+T))}\right)^{\alpha}}{1 - (1 - (1+\lambda b^2)e^{-\lambda(b^2+b)})^{\alpha}}\right\}. \tag{7.3.8}$$

Substitute these two results in Eq.(7.2.2), we get total expected replacement cost as

$$\begin{aligned} C_2(T) &= c_f \left(1 - \left\{\frac{1 - \left(1 - (1+\lambda(b+T)^2)e^{-\lambda((b+T)^2+(b+T))}\right)^{\alpha}}{1 - (1 - (1+\lambda b^2)e^{-\lambda(b^2+b)})^{\alpha}}\right\}\right) \\ &+ c_r \left\{\frac{1 - \left(1 - (1+\lambda(b+T)^2)e^{-\lambda((b+T)^2+(b+T))}\right)^{\alpha}}{1 - (1 - (1+\lambda b^2)e^{-\lambda(b^2+b)})^{\alpha}}\right\}. \end{aligned} \tag{7.3.9}$$

The total expected cycle length is

$$
\begin{aligned}
& T\frac{1-\left(1-(1+\lambda(b+t)^2)e^{-\lambda((b+t)^2+(b+t))}\right)^{\alpha}}{1-(1-(1+\lambda b^2)e^{-\lambda(b^2+b)})^{\alpha}} \\
& + \int_0^T t\alpha e^{-\lambda((b+t)^2+(b+t))}\left(1-(1+\lambda(b+t)^2)e^{-\lambda((b+t)^2+(b+t))}\right)^{\alpha-1} \\
& \left[\frac{(\lambda(1+\lambda(b+t)^2)(2(b+t)+1)-2\lambda(b+t))}{1-(1-(1+\lambda(b+t)^2)e^{-\lambda((b+t)^2+(b+t))})^{\alpha}}\right] \\
& \times\frac{1-\left(1-(1+\lambda(b+t)^2)e^{-\lambda((b+t)^2+(b+t))}\right)^{\alpha}}{1-(1-(1+\lambda b^2)e^{-\lambda(b^2+b)})^{\alpha}}\;dt \\
& = \int_0^T \bar{F}_b(t)\;dt. \qquad (7.3.10)
\end{aligned}
$$

Hence, from (7.3.6), (7.3.9) and (7.3.10) the long-run average cost per unit time is

$$
\begin{aligned}
C(b,T) = & \frac{\left(c_o + c_s\int_0^b \frac{\alpha e^{-\lambda(t^2+t)}(\lambda(1+\lambda t^2)(2t+1)-2\lambda t)\left(1-(1+\lambda t^2)e^{-\lambda(t^2+t)}\right)^{\alpha-1}}{1-\left(1-(1+\lambda t^2)e^{-\lambda(t^2+t)}\right)^{\alpha}}\;dt\right)}{\int_0^T\left(1-(1-(1+\lambda(b+t)^2)e^{-\lambda((b+t)^2+(b+t))})^{\alpha}\right)\;dt} \\
& \times\left(1-\left(1-(1+\lambda b^2)e^{-\lambda(b^2+b)}\right)^{\alpha}\right) \\
& + \frac{c_f\left(1-\left(1-(1+\lambda b^2)e^{-\lambda(b^2+b)}\right)^{\alpha}\right)}{\int_0^T\left(1-(1-(1+\lambda(b+t)^2)e^{-\lambda((b+t)^2+(b+t))})^{\alpha}\right)\;dt} \\
& - \frac{c_f\left(1-\left(1-(1+\lambda(b+T)^2)e^{-\lambda((b+T)^2+(b+T))}\right)^{\alpha}\right)}{\int_0^T\left(1-(1-(1+\lambda(b+t)^2)e^{-\lambda((b+t)^2+(b+t))})^{\alpha}\right)\;dt}
\end{aligned}
$$

$$+\frac{c_r\left(1-\left(1-(1+\lambda(b+T)^2)e^{-\lambda((b+T)^2+(b+T))}\right)^{\alpha}\right)}{\int_0^T\left(1-(1-(1+\lambda(b+t)^2)e^{-\lambda((b+t)^2+(b+t))})^{\alpha}\right)dt}$$

7.4 Summary

We discussed about burn-in process. Expressions for obtain optimal Burn-in time and optimal age under age replacement policy are derived for WL and GXE distributions.