Deepthi K.S. “Modelling and analysis of reliability data using bathtub shaped failure rate distributions.” Thesis. Research & Post Graduate Department of Statistics, St. Thomas’ College (Autonomous), University of Calicut, 2020.

CHAPTER 8

CONCLUSION AND FUTURE WORK

8.1 Conclusion

The theory of reliability is well established scientific discipline with its own principles and methods of problem solving. Probability theory and mathematical statistics play an important role in most problems in reliability theory. Many authors discussed the benefits of ageing properties like increasing failure rate, decreasing failure rate, etc. The bathtub failure rate distributions are widely used to address the lifetime inference and testing problems. The bathtub shape is 'characteristic of the failure rate curve of many well designed products and components including the human body'. Many bathtub shaped failure rate distributions are available in literature, but some of them are appropriate for given data. So identification of distribution or failure rate model shall lead to more accurate probability computations. In the present work we tried to enhanced the application of bathtub shaped failure rate distribution, proposed new BFR and UBFR distributions, compared existing bathtub shaped failure rate models, studied on the stress-strength reliability models, developed the theory and application of TTT transform in identification of BFR model of increasing convex (concave) function of lifetime random variable, and derived expression for optimal burn-in time and optimal age using proposed distributions.

In *Chapter 1*, the relevance and scope of the study and basic information about concepts are given. In *Chapter 2* presented the review of various bathtub shaped distributions.

In *Chapter 3*, two new distributions *viz.*, GXE Distribution and WL distribution have been proposed and its properties studied. We shows that these distributions exhibits bathtub shaped failure rates. For avoiding the scale problem, a three parameter WL Distribution is introduced and shown that this distribution had BFR function. The flexibility of these two proposed distributions are illustrated using two real data sets. For each data set, the proposed distributions was shown to give better fit than several other competitors exhibiting BFR function.

In *Chapter 4*, a new distribution, based on DUS transformation using Lomax distribution as baseline has been proposed and its properties are studied. This distribution exhibited UBFR function. This situation was analyzed by Efron (1988) in the context of head and neck cancer data, in which the failure rate initially increased, attended a maximum and then decreased before it finally stabilized because of a therapy. Three data sets are considered and shown that DUS-Lomax is more appropriate than some other well-known distributions (Lomax distribution, Gompertz Lomax distribution, Kumaraswamy Lomax distribution, DUS-Exponential distribution and Inverse Lindly distribution). We have shown that, DUS-Lomax has the lowest AIC, BIC, KS-Statistic, and highest Log-likelihood value and p-value. Therefore DUS-Lomax is a better alternative in situations where upside-down bathtub shaped distributions occur.

In *Chapter 5*, we compare two methods of estimating $R = P(Y < X)$ in two cases. First, when Y and X both follow three parameter generalized Lindley distribution. Second, when Y and X follows Power Lindley distribution and three parameter generalized Lindley distribution. We provide MLE procedure to estimate the unknown parameters and use this to estimate of R. Also obtain asymptotic $100(1 - \nu)\%$ CI for the reliability parameter. The simulation results indicate that MLE in the average bias and average MSE for different choices of the parameters.

Whereas to estimate the multi-component stress-strength reliability in two cases. First, when Y and X both follow three parameter generalized Lindley distribution. Second, when Y and X follows Power Lindley distribution and three parameter generalized Lindley distribution. We provide MLE procedure to estimate the unknown parameters and use this to estimate of $R_{s,k}$. Also obtain asymptotic CI for the reliability parameter. The simulation results indicate that MLE in the average bias and average MSE for different choices of the parameters. When increasing the sample sizes, MSE caused by the estimates are nearer to zero by extensive simulation.

Two sets of data are considered and shown that TPGL and PL distributions are fit with this data. For analysis, the stress-strength reliability result for the case 1 is a value of 0.2388, and for case 2, the stress-strength reliability result is 0.012, which means there is a small chance that X is greater than Y. For case 1, when $(s, k) = (1, 3)$ and $(2, 4)$ corresponding multi-component stress-strength reliability value are 0.7705 and 0.6257 respectively, and for case 2 also when $(s, k) = (1, 3)$ and $(2, 4)$ corresponding multi-component stress-strength reliability value are 0.7712 and 0.6253 respectively, which means there is a high chance that X is greater than Y. If we consider $(s, k) = (3, 5)$ corresponding MSS are for case 1 is 0.5268 and for case 2 is 0.5253, which means there is a equal chance that X is greater than Y.

In *Chapter 6*, we defined the increasing convex (concave) TTT transformation. The procedure for identifying the failure rate model is illustrated with example. If we take lifetime random variables which follows bathtub shaped failure rate distribution and we need to check whether the result is same when we apply concave and convex transformations. If yes, can the same distribution be used to fit $g(X)$. Those reasons are explained hereby using failure rate patterns. ICXTTT (ICVTTT) transform can be used for estimating the dispersion parameters, of censored data.

Finally, we discussed the burn-in process and maintenance policy often used in field work. Optimal Burn-in time and optimal age under age replacement policy for WL and GXE distributions can be obtained using the derived expression.

8.2 Future Work

On the basis of the present study some important questions are: If the distribution of system lifetime is Bathtub shaped failure rate model, identification of change points (from decreasing to constant and constant to increasing), is crucial to the system engineering. In industry burn in process, the change point estimation is very important to separate week and strong components, before send into market. We can answer questions like, how long the burn-in process needs to be continued? What is the period of useful life? etc. In inventory theory, the number of inventory to be kept for repair or replacement can be decided according to the failure behavior.

Possible future works are to (i) provide a review of known upside-down bathtub shaped distributions; (ii) develop the problem of classical and Bayesian estimation of stress-strength reliability life distribution based on upper record values; (iii) develop time-dependent stress-strength reliability models subject to random stresses at random time cycles. Each run of the system changes the power of the system over time; (iv) examine ICXTTT transformation ordering and express the hazard ordering, likelihood ratio ordering and mean residual ordering, their mutual relationships and expressions for the ICXTTT transform in terms of these ordering; (v) derive the upper and lower bounds for the optimal burn-in time. It is also desirable to study on MCMC methods for censored data, regression issues with covariates.