CHAPTER 1

Introduction and Review of Literature

1.1 Introduction

A large number of statistical applications have been seen in reliability theory and survival analysis using various lifetime distributions. However, existing distributions do have limitations when applied to certain data due to their inability to provide a proper fit. To obtain precise probability results, it is necessary to use more appropriate statistical lifetime distributions. Through the study of new statistical distributions for existing data, a fresh perspective can be provided on reliability theory and survival analysis by introducing more flexible models and an in-depth analysis of their characteristics and properties. If a better distribution is available than existing distributions, a researcher would have selected the better model. Fitness for the given data is one of the criteria for selecting the better model. If a distribution is more fitting to the given data, researchers have to leave the existing distribution and use a new model.

In reliability and survival analysis, the general structure of the system could be series or parallel. Therefore, it is essential to investigate the distributional properties of series or parallel systems when components are distributed according to any lifetime distribution available in the literature. The DUS transformation of a lifetime distribution was introduced by Kumar et al. (2015). The name 'DUS' is given according to the first letter of the author's name: Kumar, D., Singh, U., and Singh, S. K.; see Kumar et al. (2015). A DUS transformation provides a new distribution, using some baseline lifetime distribution, without adding new parameters. As far as inference is concerned, the DUS transformation is a good approach since it does not add any new parameters. Increasing the number of parameters leads to complexity in estimation and a lack of consistency. Suppose that the lifetimes of the components are distributed as a DUS-transformed distribution. What would be the distribution of a parallel system consisting of such components? This question has to be answered through research. Several statistical distributions are available, like exponential, Weibull, Lomax, Gamma, etc., for reliability and survival analysis. Also, a large number of transformations are available in the literature, like the DUS transformation, the Kavya-Manoharan (KM) transformation, etc. They are used in reliability analysis. Their performance in engineering systems like series systems and parallel systems is to be investigated in detail.

Mixture distributions are useful when dealing with lifetime data analysis. When a new component switches on for the first time, it may fail at the same moment, or it may fail while working due to overvoltage, jerking, or any such shocks, or it may fail due to degradation. So failure due to random shocks or random failure can be modelled using an exponential distribution. Since there is a chance of failure due to the degradation of components, such failure time may be distributed as any other lifetime distribution if it is fitted to the data. When both situations happen to a group of components installed for a mission, the researcher should use a mixture distribution. Investigation of certain mixtures is essential to knowing how they behave in reliability and survival analysis. Moreover, the estimation of parameters also has to be studied in detail.

Stress-strength reliability analysis using statistical distributions is very important in the fields of engineering, quality control, and medicine. According to stress-strength reliability, a network's ability to withstand stress is the measure of its reliability or safety. It is quite desirable to estimate stress-strength reliability (R) using more appropriate models, especially mixture models. Investigation of the statistical properties of reliability estimators is also imperative when dealing with inferential procedures. This stress-strength parameter, R, measures the performance of systems in the context of mechanical reliability. Apart from the inferential information, R provides the probability of system failure if the system fails while the applied stress exceeds its strength. The field of reliability engineering places a great deal of emphasis on accelerated life testing, in particular, step-stress accelerated life testing. The inference of step stress models based on various statistical distributions is useful for accelerated life testing.

Birnbaum-Saunders (BS) distribution is one of the important distributions when dealing with fatigue failure. Several generalizations of the BS distribution are available in the literature. The usefulness of its generalizations has to be explored more. Due to the complexity of the model, the estimation of parameters becomes more complicated. A detailed study on the estimation of parameters and the estimation of the confidence interval is required for the generalized BS models.

In mathematical statistics, reliability theory and survival analysis examine a specific class of time-to-event random variables. Methods for evaluating and forecasting a product's successful operation or performance are discussed in reliability analysis. Due to rapid advances in technology, the development of highly sophisticated products, intense global competition, and rising customer expectations, manufacturers are under increased pressure to produce high-quality, reliable products. Customers expect engineering products to be reliable and safe when they purchase them. For a substantial time period, systems like vehicles, machines, telecommunication devices, power generation systems, and so forth should be capable of performing their intended functions under normal operating conditions.

In technical terms, reliability can be defined as a system's ability to perform its intended mission within a specified time under normal operating conditions. Enhancing the reliability of products is one of the most important aspects of improving the quality of products. Methods of reliability have been developed and applied to enhance the safety and reliability of complex technological systems, such as nuclear power systems, chemical plants, space systems, hazardous waste facilities, and offshore installations. Survival analysis is generally defined as a set of methods for analyzing data where the outcome variable is the time until an event of interest occurs. It can be death, the onset of a disease, failure, or the completion of a mission. The time to event or survival time may be measured in days, hours, weeks, years, etc.

Reliability theory and survival analysis mainly focus on positive random variables, often called lifetimes. The distribution function provides a complete characterization of a lifetime random variable. How effectively a lifetime is understood depends on failure, death, or some other "end event".

By utilizing appropriate statistical distributions for modeling a lifetime, reliability calculations can be simplified and made more accurate. By analyzing the distribution of the lifetime of the system, reliability, maintenance, and replacement measures can be planned accordingly. An analysis of reliability requires the identification of the failure rate model to ascertain which distribution is suitable for the available data. The distributions available in the literature are often insufficient to explain the distributional characteristics and reliability analysis of the data given. Thus, researchers are constantly forced to come up with more suitable distributions for the data that they are provided with.

1.2 Basic Concepts

1.2.1 Reliability or Survival Function

When it comes to probability, an object's reliability is defined as the probability that it will carry out its intended function for a predetermined timeframe while working under conventional environmental conditions. This probability is referred to as the survival probability in survival analysis.

The probability that an object will survive to time t is defined by the reliability function (or survival function) of the lifetime variable, T, indicated by $R(t)$ (or $S(t)$) where,

$$R(t) = S(t) = P[T > t] = \int_t^\infty f(x)dx.$$

Furthermore, $R(t) = 1 - F(t)$, where $F(.)$ is the cumulative density function (CDF). This feature is often used in reliability analysis.

1.2.2 Failure Rate or Hazard Rate Function

The failure rate function, or hazard rate function (HRF), denoted by $h(t)$, can be considered as the probability that an object will fail in the interval $(t, t + \Delta t)$ for small Δt, assuming that it hasn't already failed before t. It is referred to as the ratio of the probability density function (PDF) to the survival function and is given by

$$h(t) = \lim_{\Delta t \to \infty} \frac{P[t < T \leq t + \Delta t | T > t]}{\Delta t} = \frac{f(t)}{R(t)}.$$

Most applications will result in a reduction in the lifetime of the system beyond the specified age, which is a reasonable assumption. In other words, the survival rate of a system decreases with age. When units exhibit this behavior, their life distributions are considered positive ageing distributions.

Ageing is an important concept in understanding which real-life distributions are suitable for reliability data analysis. An ageing concept largely describes how a device ages with time. Though in most cases, ageing has an adverse effect on a product, there are some other cases in which ageing is beneficial. Ageing has a direct impact on the behavior of the HRF. They can be used in maintenance planning, replacement planning, resource allocation, etc. Using the HRF, one can conveniently define ageing.
The lifetime distributions can be classified into the following categories based on the HRF.

- **Constant Failure Rate (CFR)**
  A constant failure rate is observed during the midlife stage because failures mostly occur as a result of external factors or random failures. In most cases, this period is referred to as the "working life" of a system or component because most systems spend the majority of their lifetimes in this stage.

- **Increasing Failure Rate (IFR)**
  The concept of IFR is intuitively based on the deterioration of components. In the context of failure rates, an IFR is one in which $h(x)$ increases monotonically over x or equivalently, when $-logF(x)$ is convex.

- **Decreasing Failure Rate (DFR)**
  A DFR refers to a process where the probability of an event occurring in the future declines with time. When earlier failures are removed or corrected, there is a DFR during the "infant mortality" period. This corresponds to a situation where the HRF is descending. An improvement in DFR occurs as the system ages. In the context of failure rates, an DFR is one in which $h(x)$

decreases monotonically over x or equivalently, when $-logF(x)$ is concave.

- **Bathtub Shaped Failure Rate (BFR)**
  If the HRF of F decreases initially and then remains constant for a period, and then increases over time, then corresponding distributions are called BFR distributions. BFR refers to the early life, useful life, and wear-out phases of a component or system.

  In the case of humans, failures typically occur during the early life period as a result of birth defects. Failures occurring during useful life can be referred to as chance failures. As the unit ages, the more likely it is to fail in the wear-out region.

  Lifetime distributions with BFR are an important class of lifetime distributions since the lifetime of electronic, electromechanical, and mechanical products is often modeled using them. According to survival analysis, human life exhibits this pattern.

- **Upside-Down Bathtub Shaped Failure Rate (UBFR)**
  A UBFR distribution is characterized by a failure rate $h(x)$, which increases initially for $x \in (0, x_0)$, then becomes constant, and finally decreases for $x > x_0$.

The characterization of distributions, whether IFR or DFR, or BFR reduces the selection of the model in reliability analysis. The bathtub shape is characteristic of the failure rate curve of many well-designed products and components including the human body. Monotonic ageing concepts are found to be popular among many reliability engineers. However, in many practical applications, the effect of age is initially beneficial, but after a certain period, its age-adverse indication is positive.

1.2.3 Some Statistical Distributions

The lifetime distributions used in this thesis are given below.

Exponential Distribution

The exponential distribution is a continuous distribution related to the length of time between events. The exponential distribution was the first to be widely used. In addition to its simple representation of CDF, PDF, and HRF, as well as its availability of simple statistical methods for data analysis, it is also an effective method for predicting the lifetime of many types of manufactured items. While the constant hazard rate may be useful to some extent when considering the use of this distribution, caution should be exercised when considering its use. This is because inferential procedures may be sensitive to deviations from the exponential distribution when utilizing it. Furthermore, this distribution has a lack of memory property. It has the PDF

$$f(t|\mu) = \mu e^{-\mu t}, \ \ t > 0, \mu > 0.$$

The corresponding CDF and HRF are given, respectively, by

$$F(t|\mu) = 1 - e^{-\mu t}, \ \ t > 0, \mu > 0,$$

and

$$h(t|\mu) = \mu, \ \ \mu > 0.$$

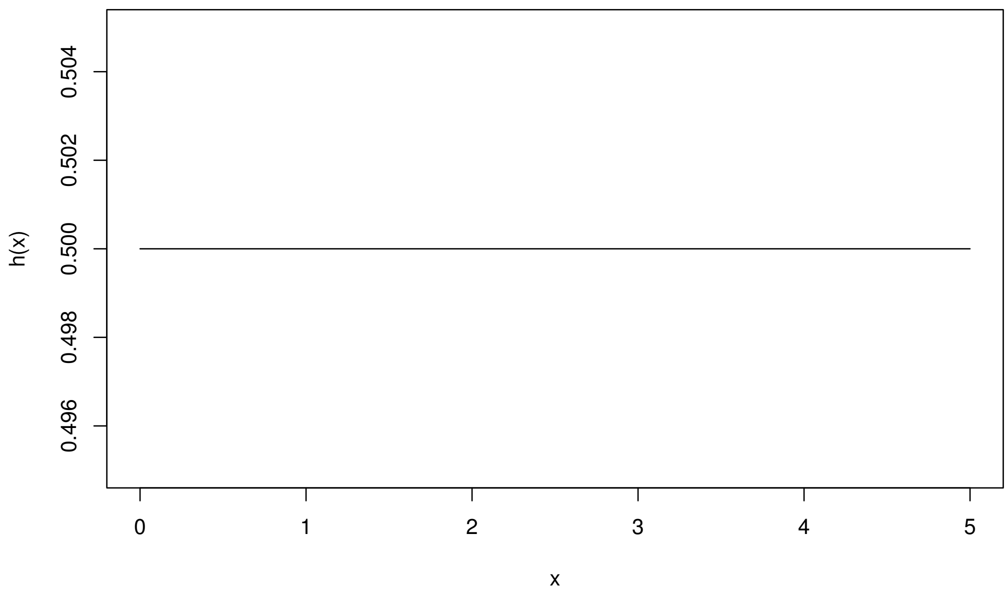

Figure 1.1: Exponential hazard function plot

This distribution has a mean $\frac{1}{\mu}$ and a variance $\frac{1}{\mu^2}$. The standard exponential distribution is defined as the distribution where μ equals 1.

Gamma Distribution

The gamma distribution models the right-skewed data and is one of the most commonly used continuous distributions. As a result of the fame of the exponential distribution, it has become prevalent to use the gamma distribution as a model for the sum of lifetimes with an exponential distribution. In addition to being naturally derived from the convolution of exponential distributions, the gamma life distribution has the disadvantage that it cannot be algebraically treated. It has the PDF

$$f(t|\lambda, \theta) = \frac{\theta^{\lambda}}{\Gamma(\lambda)} t^{\theta-1} e^{-\theta t},\ t > 0, \lambda > 0, \theta > 0.$$

The CDF is given by

$$F(t|\lambda, \theta) = \frac{\gamma(\lambda, \theta t)}{\Gamma(\lambda)},$$

where $\gamma(\lambda, \theta t)$ is the incomplete gamma function. The mean and variance of gamma

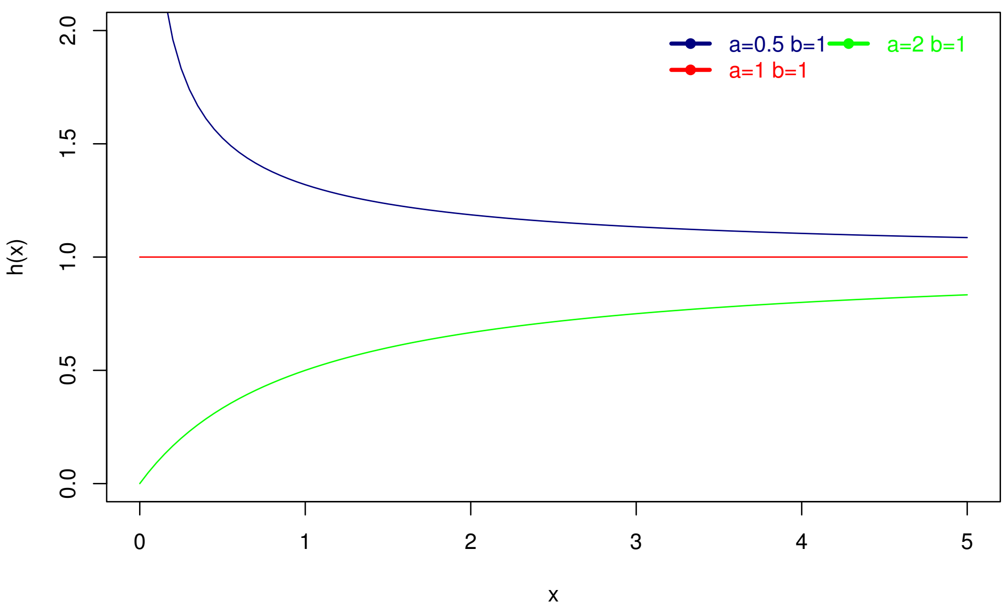

Figure 1.2: Gamma hazard function plot

distribution are $\lambda\theta$ and $\lambda\theta^2$, respectively. The skewness of the gamma distribution only depends on its shape parameter, λ, and it is equal to $2/\sqrt{\lambda}$. In the gamma distribution, HRF is IFR for $\lambda > 1$, DFR for $\lambda < 1$, and CFR for $\lambda = 1$, see Figure 1.2.

Weibull Distribution

Weibull distribution is perhaps the most widely used lifetime distribution. It is convenient to describe different types of hazards using the Weibull distribution, as it is flexible in describing them and mathematically manageable. As a result of its flexibility, the tool is used in a wide range of settings, including quality control, reliability analysis, medical research, and engineering applications. It's PDF is given by

$$f(t|\kappa,\nu) = \kappa\,\nu\,t^{\kappa-1}\,e^{-\nu t^{\kappa}}, t>0, \kappa>0, \nu>0.$$

The corresponding CDF and HRF are given by

$$F(t|\kappa,\nu) = 1 - e^{-\nu t^{\kappa}},\ t>0,$$

and

$$h(t|\kappa,\nu) = \kappa\,\nu\,t^{\kappa-1},\ t>0$$

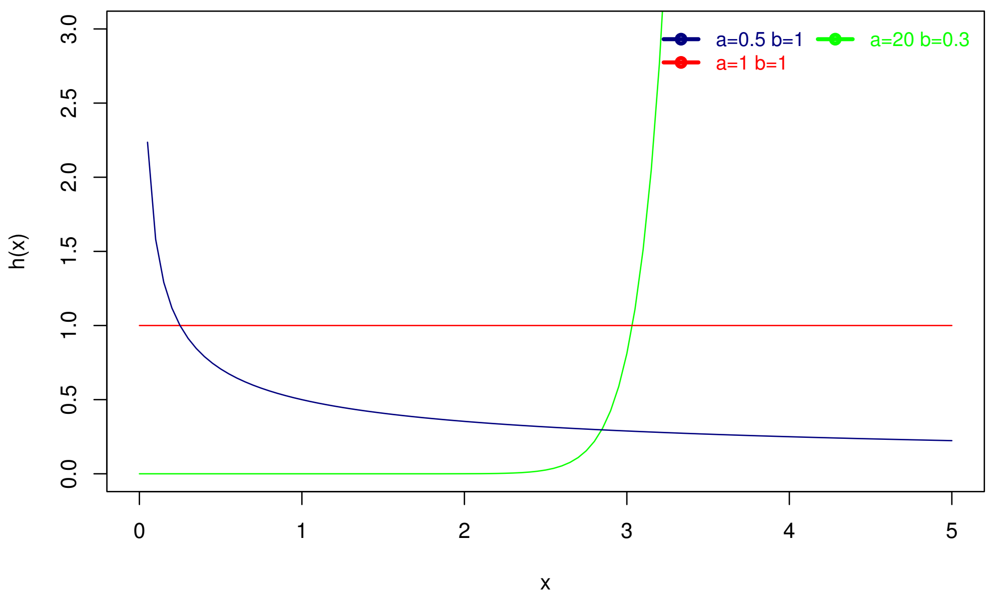

Figure 1.3: Weibull hazard function plot

The mean and variance of this distribution is $\frac{\Gamma(1+1/\kappa)}{\nu^{1/\kappa}}$ and $\frac{1}{\nu^{1/\kappa}}[\Gamma(1+2/\kappa)+\Gamma(1+1/\kappa)^2]$. It may be mentioned here that the Weibull distribution can be positively or negatively skewed depending upon the value of the shape parameter κ. The Weibull distribution demonstrates DFR for $\kappa<1$, IFR for $\kappa>1$, and CFR for $\kappa=1$, see Figure 1.3.

This distribution can be used quite conveniently for censored data as well.

Lomax Distribution

Based on business failure lifetime data, Lomax (1954) developed the Lomax distribution, which has a heavily skewed distribution. Distributions such as this one, a shifted Pareto distribution, are widely used in survival analysis and have many applications in actuarial science, reliability theory, business, network analysis, economics, operations research, medical science, and internet traffic modeling, among others. The PDF of the Lomax distribution has the form

$$f(t|k,\lambda) = \frac{k}{\lambda}\left(1+\frac{t}{k}\right)^{-(\lambda+1)},\ t>0,\ k>0,\ \lambda>0,$$

and the CDF is given by

$$F(t|k,\lambda) = 1-\left(1+\frac{t}{k}\right)^{-\lambda},\ t>0,\ k>0,\ \lambda>0.$$

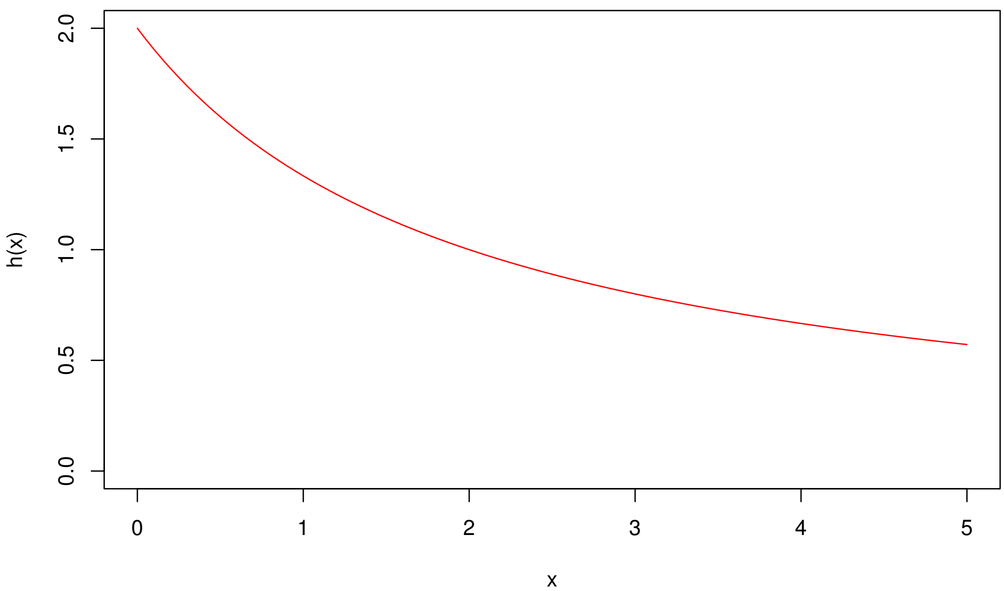

Figure 1.4: Lomax hazard function plot

The mean function is given by

$$E(T) = \frac{k}{\lambda-1}\ for\ \lambda>1,$$

and the variance function is

$$V(T) = \frac{\lambda k^2}{(\lambda-1)^2(\lambda-2)}, \; for\, \lambda > 2.$$

The HRF always has a DFR property associated with it, see Figure 1.4.

Birnbaum Saunders Distribution

- **Univariate Birnbaum Saunders Distribution**

  A random variable following the BS distribution is defined through a standard normal random variable. Therefore the PDF and CDF of the BS model can be expressed in terms of the PDF and CDF of the standard normal distribution. The CDF of a two parameter BS random variable T for $\alpha > 0$ and $\beta > 0$ can be written as

$$F_T(t|\alpha,\beta) = \Phi\left[\frac{1}{\alpha}\left(\left(\frac{t}{\beta}\right)^{\frac{1}{2}} - \left(\frac{\beta}{t}\right)^{\frac{1}{2}}\right)\right], \; t > 0, \tag{1.2.1}$$

  where $\Phi(.)$ is the standard normal CDF. The PDF of BS distribution is

$$f_T(t|\alpha,\beta) = \begin{cases} \frac{1}{2\alpha\beta\sqrt{2\pi}}\left[\left(\frac{\beta}{t}\right)^{\frac{1}{2}} + \left(\frac{\beta}{t}\right)^{\frac{3}{2}}\right] e^{\left[-\frac{1}{2\alpha^2}\left(\frac{t}{\beta}+\frac{\beta}{t}-2\right)\right]} & \text{if } t > 0 \\ 0 & \text{otherwise} \end{cases} \tag{1.2.2}$$

  Here $\alpha > 0$ and $\beta > 0$ are the shape and scale parameters respectively. The BS distribution has DFR and UBFR for its HRF, see Figure 1.5.

- **Bivariate Birnbaum Saunders Distribution**

  The bivariate Birnbaum-Saunders (BVBS) distribution was introduced by Kundu et al. (2010). The bivariate random vector (T_1, T_2) is said to have a BVBS distribution with parameters $\alpha_1, \beta_1, \alpha_2, \beta_2$, and ρ if the CDF of (T_1, T_2) can be expressed as

$$F(t_1,t_2) = \Phi_2\left[\frac{1}{\alpha_1}\left(\sqrt{\frac{t_1}{\beta_1}} - \sqrt{\frac{\beta_1}{t_1}}\right), \frac{1}{\alpha_2}\left(\sqrt{\frac{t_2}{\beta_2}} - \sqrt{\frac{\beta_2}{t_2}}\right)\right] \tag{1.2.3}$$

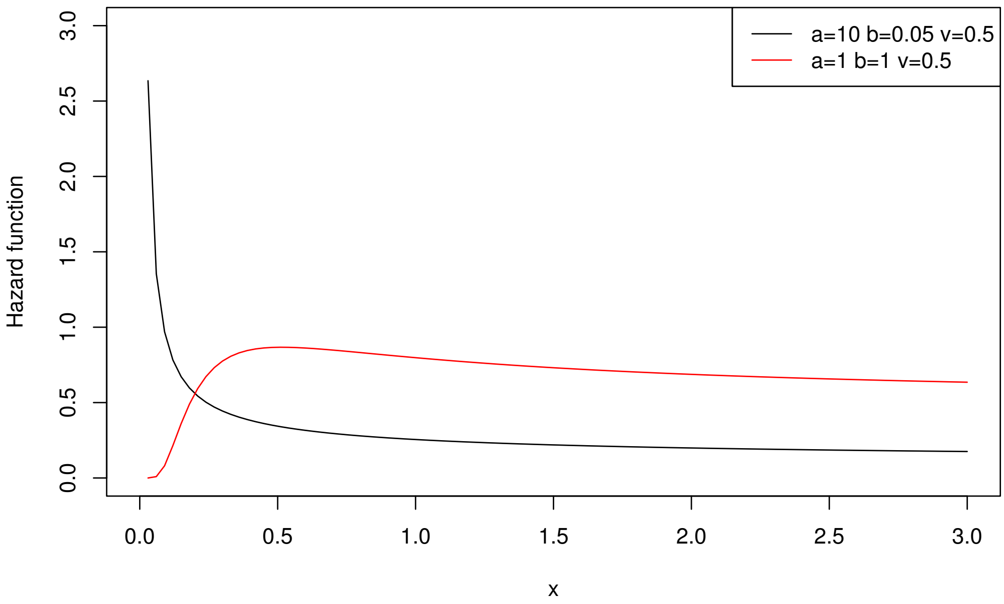

Figure 1.5: Birnbaum Saunders hazard function plot

for $t_1 > 0$, $t_2 > 0$, $\alpha_1 > 0$, $\beta_1 > 0$, α_2, $\beta_2 > 0$, and $-1 < \rho < 1$. Here $\Phi_2(u, v; \rho)$ is the CDF of standard bivariate normal vector (Z_1, Z_2) with correlation coefficient ρ. The PDF corresponding to Eq.(1.2.3) is

$$f(t_1,t_2) = \frac{1}{8\pi\alpha_1\alpha_2\beta_1\beta_2\sqrt{1-\rho^2}}\left[\left(\frac{\beta_1}{t_1}\right)^{\frac{1}{2}} + \left(\frac{\beta_1}{t_1}\right)^{\frac{3}{2}}\right]\left[\left(\frac{\beta_2}{t_2}\right)^{\frac{1}{2}} + \left(\frac{\beta_2}{t_2}\right)^{\frac{3}{2}}\right]$$
$$e^{-\frac{1}{2(1-\rho^2)}\left[\frac{1}{\alpha_1^2}\left(\sqrt{\frac{t_1}{\beta_1}}-\sqrt{\frac{\beta_1}{t_1}}\right)^2+\frac{1}{\alpha_2^2}\left(\sqrt{\frac{t_2}{\beta_2}}-\sqrt{\frac{\beta_2}{t_2}}\right)^2-\frac{2\rho}{\alpha_1\alpha_2}\left(\sqrt{\frac{t_1}{\beta_1}}-\sqrt{\frac{\beta_1}{t_1}}\right)\left(\sqrt{\frac{t_1}{\beta_1}}-\sqrt{\frac{\beta_1}{t_1}}\right)\right]}$$

for $t_1 > 0$, $t_2 > 0$, $\alpha_1 > 0$, $\beta_1 > 0$, α_2, $\beta_2 > 0$, and $-1 < \rho < 1$.

- **Multivariate Birnbaum Saunders Distribution**

Kundu et al. (2013) introduced the multivariate BS distribution. Let $\underline{\alpha}, \underline{\beta} \in \mathbb{R}^p$, where $\underline{\alpha} = (\alpha_1, \cdots, \alpha_p)^T$ and $\underline{\beta} = (\beta_1, \cdots, \beta_p)^T$, with $\alpha_i > 0$, $\beta_i > 0$ for $i = 1, \cdots, p$. Let $\mathbf{\Gamma}$ be a $p \times p$ positive-definite correlation matrix. Then, the random vector $\underline{T} = (T_1, \cdots, T_p)^T$ is said to have a p-variate BS distribution with parameters $(\underline{\alpha}, \underline{\beta}, \mathbf{\Gamma})$ if it has the joint CDF as

$$
\begin{aligned}
P(\underline{T} \le \underline{t}) &= P(T_1 \le t_1, \cdots, T_p \le t_p) && (1.2.4)\\
&= \Phi_p\left[\frac{1}{\alpha_1}\left(\sqrt{\frac{t_1}{\beta_1}} - \sqrt{\frac{\beta_1}{t_1}}\right), \cdots, \frac{1}{\alpha_p}\left(\sqrt{\frac{t_p}{\beta_p}} - \sqrt{\frac{\beta_p}{t_p}}\right); \mathbf{\Gamma}\right] && (1.2.5)
\end{aligned}
$$

for $t_1 > 0, \cdots, t_p > 0$. Here, for $\underline{u} = (u_1, \cdots, u_p)^T$, $\Phi_p(\underline{u}; \mathbf{\Gamma})$ denotes the joint CDF of a standard normal vector $\underline{Z} = (Z_1, \cdots, Z_p)^T$ with correlation matrix $\mathbf{\Gamma}$. The joint PDF of $\underline{T} = (T_1, \cdots, T_p)^T$ can be obtained from the above equation as

$$
\begin{aligned}
f_{\underline{T}}(\underline{t}; \underline{\alpha}, \underline{\beta}, \mathbf{\Gamma}) = \phi_p&\left(\frac{1}{\alpha_1}\left(\sqrt{\frac{t_1}{\beta_1}} - \sqrt{\frac{\beta_1}{t_1}}\right), \cdots, \frac{1}{\alpha_p}\left(\sqrt{\frac{t_p}{\beta_p}} - \sqrt{\frac{\beta_p}{t_p}}\right); \mathbf{\Gamma}\right)\\
&\times \prod_{i=1}^{p} \frac{1}{2\alpha_i\beta_i}\left\{\left(\frac{\beta_i}{t_i}\right)^{\frac{1}{2}} + \left(\frac{\beta_i}{t_i}\right)^{\frac{3}{2}}\right\},
\end{aligned}
$$

for $t_1 > 0, \cdots, t_p > 0$; here, for $\underline{u} = (u_1, \cdots, u_p)^T$,

$$
\phi_p(u_1, \cdots, u_p; \mathbf{\Gamma}) = \frac{1}{(2\pi)^{\frac{p}{2}}|\mathbf{\Gamma}|^{\frac{1}{2}}} exp\{-\frac{1}{2}\underline{u}^T\mathbf{\Gamma}^{-1}\underline{u}\}
$$

is the PDF of the standard normal vector with correlation matrix $\mathbf{\Gamma}$.

Type-II Gumbel Distribution

Type II Gumbel distribution is one of the statistical distributions that are used to model extreme values. The PDF of Type-II Gumbel distribution is

$$
f(t|\beta, \theta) = \beta\theta t^{\beta-1}e^{-\theta t^{-\beta}},\ t > 0,\ \beta > 0,\ \theta > 0,
$$

and the CDF is given by

$$
F(t|\beta, \theta) = e^{-\theta t^{-\beta}},\ t > 0,\ \beta > 0,\ \theta > 0.
$$

The Weibull distribution is produced when $\theta = \mu^{-\kappa}$ and $\beta = -\kappa$ are substituted. The mean and variance of Type-II Gumbel distribution are $\theta^{\frac{1}{\beta}}\Gamma(1-\frac{1}{\beta})$ and $\theta^{\frac{2}{\beta}}(\Gamma(1-\frac{1}{\beta}) - \Gamma(1-\frac{1}{\beta})^2)$, respectively. This is always an asymmetric distribution. The HRF plot of this distribution shows both DFR and UBFR, see Figure 1.6.

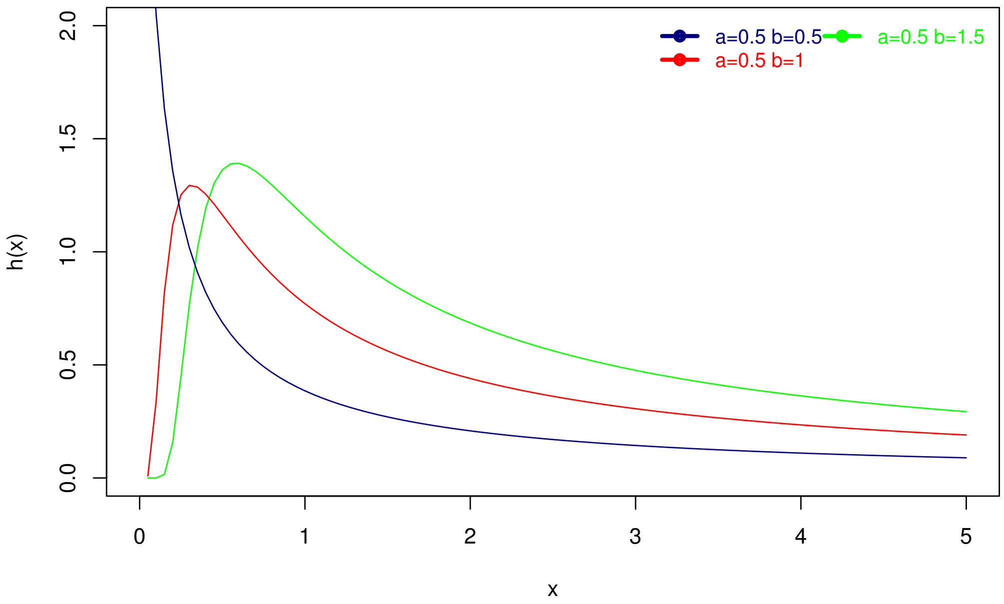

Figure 1.6: Type-II Gumbel hazard function plot

1.2.4 DUS Transformation

DUS (Dinesh-Umesh-Sanjay) transformation is a transformation method used to generate new lifetime distributions proposed by Kumar et al. (2015). In terms of computation and interpretation, this transformation produces a parsimonious result since it does not include any new parameters other than those involved in the baseline distribution.

In the case where $F(x)$ is the CDF of the baseline distribution, then the CDF of the new DUS transformed distribution is as follows:

$$G(x) = \frac{1}{e-1}[e^{F(x)} - 1].$$

Then the PDF becomes

$$g(x) = \frac{1}{e-1}[e^{F(x)} f(x)],$$

where $f(x)$ is the PDF of the baseline distribution.

1.2.5 Censoring

In reliability theory and survival analysis, censoring is a frequently used concept. One scenario involving censored data would be if the study was terminated before all sample items failed. As the investigation has already ended, it is unknown when the remaining parts will fail. As a result, the sample has two sets of observations: one with real failure times, known as complete data (uncensored), and the other with mere constraints on failure times, known as incomplete data (censored data).

Type I Censoring

The most frequently used censoring method in reliability engineering is Type I censoring. The experiment is terminated by Type-I censoring at a predefined time T. For example, in life testing experiment, n items are placed on a test, but lifetimes will be known only for the items that fail by time T. Therefore, according to this scheme, the duration of the experiment is fixed, but the number of failures is random. It has a major advantage that if there are very few failures, statistical analysis will be inefficient.

Type II Censoring

Under the Type II censoring scheme, mth ($m \leq n$) failure terminates the experiment. In a life-testing experiment, n items are tested. Test termination occurs at the time of the mth failure instead of waiting until all n observations have failed. There may be instances in which it takes a long time for the test to fail all n items. A test of this type can be cost-effective and time-saving. There is a fixed number of observed failures in this censoring scheme, but the test duration is random.

1.2.6 Stress-Strength Reliability

A stress-strength (SS) model, which is largely used in reliability engineering but is also utilized in economics, quality control, psychology, and medicine, compares the strength and stresses of a system. Both stresses and strength are viewed as distinct random variables in a SS model.

As a result of technological advancements, a variety of fields have become increasingly concerned with the issue of enhancing network reliability in the modern world. Despite receiving a certain amount of stress, some products can withstand it due to their strength. Appliances, however, tend to malfunction if more stress than strength is given. Assume that Y represents the random stress placed on a certain appliance and X represents the random strength needed to withstand the force. The device will fail if and only if the applied stress ever exceeds its threshold level. Therefore, $R = P(X > Y)$ gives the reliability of a system as a measure.

Component reliability in the SS environment is determined by the PDF $g_1(x)$ of the strength of the unit or system, X, and the PDF $g_2(y)$ of the stress Y. At any given moment, the system will fail if the applied stress exceeds its strength. In the case where X and Y are independently distributed, the SS reliability of the component can be calculated as follows:

$$R = P(X > Y) = \int_0^{\infty} \left[\int_y^{\infty} g_1(x)dx \right] g_2(y)dy.$$

1.2.7 Accelerated Life Testing

In an accelerated life test, failure rates are accelerated by subjecting the component to more stress. It follows that the failure time is determined by the stress factor and that higher levels of stress will result in a quicker failure time. At higher temperatures, some components may fail more rapidly; however, at lower temperatures, they may be more likely to last longer. The time required under low-stress conditions may not be sufficient to determine system reliability, which will be tested in conditions of increasing stress, resulting in a relatively short duration of the experiment. With this technology, failures that, in normal circumstances, would take a long time to appear can be seen sooner. In addition, the size of the data can be increased without having to spend a lot of money or time. Reliability testing of this type is known as accelerated life testing (ALT).

Step-Stress ALT

Several types of ALT exist in which stress is applied under accelerated conditions in various ways, including constant stress ALT (CSALT), step-stress ALT (SSALT), and progressive stress ALT (PSALT).

In CSALT, the stress applied to the test product is time-independent. The testing units are subjected to a constant, higher-than-usual level of stress until either all units fail or the test is terminated, resulting in censored test data. In PSALT, the stress applied to a test product continuously increases over time.

Step-stress tests are accelerated life tests in which the amount of stress applied to each unit progressively increases over time. There may be more than one stress change point in this case. According to the simple SSALT model, for example, a random sample of n units is initially placed on the low-stress level x_1 and is allowed to run until the predetermined time T_1 is reached. As soon as time T_1 is up, the stress is changed to x_2 for the remaining unfailed units. Upon changing the stress to x_2, the test continues until all units fail or are censored. Accordingly, these tests are commonly used to estimate the distribution parameters of failure times under normal operating conditions based on observed orders of failure times.

Compared to other test methods, the SSALT model has the primary advantage of reducing overall test duration. SSALT yields faster failures owing to increasing levels of stress. In ALT data analysis, we need to determine the PDF of a test until at design condition from ALT data instead of traditional life test data obtained under normal conditions. To do that, we must have an appropriate life distribution and a life-stress relationship.

1.2.8 Model Selection Criteria

The model selection criterions used in this dissertation are given below.

Kolmogorov-Smirnov Test

To determine whether or not a given sample reflects a population with a specific distribution, Kolmogorov (1933) proposed the Kolmogorov-Smirnov (KS) test. The KS test determines the difference between the estimated CDF of the distribution and the sample's empirical distribution function. In this case, the null hypothesis is H_0: The sample follows the particular distribution, and the alternative hypothesis is H_1: The sample doesn't follow the particular distribution. It is pertinent to note that when comparing more than one distribution, it is more appropriate to choose the distribution with a smaller KS value.

Cramér-Von Mises Test

Based on the sum of squared differences between the empirical distribution function and theoretical distribution function, Cramér-Von Mises (CVM) test statistic can be defined as follows:

$$CVM = \frac{1}{12n} + \sum_{i=1}^{n} \Big[F(x_i,\theta) - \frac{2i-1}{2n}\Big]^2$$

Whenever the value of the CVM test statistic exceeds the critical point, the null hypothesis is rejected.

Akaike's Information Criterion

The Akaike information criterion (AIC) measures the fit of a model to the data it was derived from. The AIC can be used in statistics to determine the most appropriate model for the data by comparing different possible models. It is considered best to select the model with the lowest AIC. In order to calculate the AIC, the following factors must be considered:

- the number of unknown parameters in the model.
- the MLE of the model.

Therefore, the AIC can be defined as

$$AIC = 2v - 2\log L,$$

where v is the number of unknown parameters in the model and L represents the maximized likelihood value.

Bayesian Information Criterion

In statistics, the Bayesian Information Criterion (BIC) is a criterion for selecting between two or more models. It is considered more appropriate to select the model with the lowest BIC. It is defined as

$$BIC = v\log(m) - 2\log(L),$$

where m is the sample size, v is the number of unknown parameters in the model, and L represents the maximized likelihood value.

Corrected Akaike's Information Criterion

For a small sample size, there's a high probability that AIC will choose models containing too many parameters, causing AIC to overfit. In order to mitigate such a risk of overfitting, AICc was introduced: corrected Akaike Information Criterion (AICc) is basically the same as AIC but with a modification for small sample sizes. The AICc is defined as

$$AICc = AIC + \frac{2v(v+1)}{m-v-1},$$

where m is the sample size and v is the number of unknown parameters in the model.

Consistent Akaike's Information Criterion

The consistent Akaike's Information Criterion (CAIC) is one of the information criterion used for selecting different models and is defined as

$$CAIC = -2\log(L) + v[\log(m) + 1],$$

where m is the sample size, v is the number of unknown parameters in the model, and L represents the maximized likelihood value.

1.3 Review of Literature

In the modern world, reliability is well-known and is expanding rapidly. Its objective is to increase the effectiveness of the system by developing new techniques. Therefore it has gained much importance among manufacturers.

1.3.1 Failure Rates

The occurrence of IFR is of interest in a wide variety of real-world systems, Koutras (2011) and Ross et al. (2005). The gamma distribution and the Weibull distribution are the most popular distributions with IFR (both distributions also exhibit DFR and CFR). Exponentiated exponential distribution and weighted exponential distribution have been introduced in place of the gamma and Weibull distributions by Gupta and Kundu (2001, 2009), respectively. Cancho et al. (2011) proposed an IFR lifetime distribution called the Poisson-exponential (PE) distribution. Instead of the Weibull, gamma, exponentiated exponential, weighted exponential, and PE distributions, Bakouch et al. (2014) presented the binomial-exponential 2 (BE2) distribution, a two-parameter lifetime distribution with IFR properties. When the sample size has a zero-truncated binomial distribution, the BE2 distribution is created as a distribution of a random sum of independent exponential random variables.

There have been many instances where the data show DFR function. According to Proschan (1963), air conditioning systems on aircraft follows DFR distribution. Kus (2007) examined earthquakes that have occurred in the North Anatolia fault zone during the last century and concluded that DFR distribution is quite accurate. Adamidis and Loukas (1998) introduced a two-parameter DFR distribution. Alpha power transformed Lindley distribution with DFR and BFR was introduced by Dey et al. (2019) with application to earthquake data.

Among the classes of life distributions that have received considerable attention is the one that exhibits bathtub-shaped failure rates. Detailed accounts of such distributions have been provided by Rajarshi and Rajarshi (1988). F is said to have BFR if its failure rate initially decreases, then remains constant for a duration, and eventually increases over time. The class of lifetime distributions featuring a BFR function is significant since the lifespans of electronic, electromechanical, and mechanical products are typically represented with this feature, as noted in Barlow and Proschan (1975). Moreover, in survival analysis, human lifetimes typically exhibit this pattern.

Kao (1959), Glaser (1980), and Lawless (1982) offer a variety of illustrations of BFR distributions. As a mixture of a group of IFR distributions for competing risk models, Hjorth (1980) portrayed BFR distributions. The BFR distributions were discussed by Lai et al. (2001), while Xie et al. (2002) investigated modified Weibull extension models that have BFR functions helpful for cost analysis and decision-making concerning reliability. Block et al. (2008) examined the continuous mixture of entire families of distributions with BFR functions. The generalized linear failure rate distribution and its characteristics were developed by Sarhan and Kundu (2009).

Mudholkar and Srivastava (1993) and Xie and Lai (1996) proposed modifications to Weibull distributions to make them suitable for BFR data. Additionally, Chen (2000) developed a two-parameter BFR model for analyzing survival data. An additive model for lifetime data with BFR was investigated by Wang (2000) based on the Burr XII distribution. The generalized Rayleigh distribution, also known as the two-parameter Burr Type X distribution, featured an IFR or BFR function and was introduced by Surles and Padgett (2005). A new exponential-type distribution with CFR, IFR, DFR, BFR, and UBFR functions was recently proposed by Lemonte (2013a) and can be utilized to simulate survival data in reliability problems and fatigue life studies. The parameter estimation of a three-parameter Weibull-related model with IFR, DFR, BFR, and UBFRs was studied by Zhang et al. (2013). Weibull extension with BFR function was obtained by Wang et al. (2014) using type-II censored samples.

Due to the ability of some generalized Gamma-type distributions to model different BFR functions, Parsa et al. (2014) studied the differences between the change points of failure rate and mean residual life functions. A novel finite interval lifetime distribution model for fitting the BFR curve was discussed by Wang et al. (2015). Shehla and Ali Khan (2016) used an exponential power model with the BFR function to study reliability analysis. In order to model fuzzy lifetime data, Shafiq and Viertl (2017) provided generalized estimators for the parameters and failure rates of the BFR distributions. A new Lindley Weibull distribution that includes unimodal and BFRs was introduced by Cordeiro et al. (2018). A generalized Weibull uniform distribution that adds DFR or BFR features to the Weibull distribution was proposed by Al-Abbasi et al. (2019). The reliability analysis of gas-turbine engines with BFR distribution was examined by Ahsan et al. (2019). Based on adaptive progressive type-II censored data, Chen and Gui (2020) studied the inferential problem of two parameters of a lifetime distribution using BFR functions. According to Deepthi and Chacko (2020a), an UBFR model can be constructed using DUS Transformation of the Lomax Distribution. Shrahili and Kayid (2022) described a generalized Pareto distribution characterized by a heavy right tail and UBFR. On the basis of a new power function, Sindhu et al. (2023) proposed a bathtub-shaped nonhomogeneous Poisson process software reliability model distribution.

1.3.2 Birnbaum-Saunders Distribution

As a result of the continuous vibration present in commercial aircraft and the problems resulting from it, Birnbaum and Saunders (1968) developed an innovative probabilistic model to describe material specimen lifetimes as a result of fatigue due to cyclical stress and tension caused by exposure to fatigue. Birnbaum and Saunders (1969b); Birnbaum and Saunders (1969a) formulated the fatigue-life distribution that would later bear their names, defining it as a life distribution and establishing an approach for estimating the parameters of this two-parameter distribution.

Unlike most other distributions, the Birnbaum-Saunders distribution is based on cumulative damage that causes fatigue in materials over time. Birnbaum and Saunders (1969a) derived the fatigue-life distribution from a simulation model, which showed that cumulative damage caused by the development and growth of a dominant crack often exceeds a threshold value and results in the failure of the specimen. Some of the assumptions made by Birnbaum and Saunders (1969a) were relaxed by Desmond (1985), strengthening the rationale for using this distribution.

There have been several attempts to extend and generalize the BS distribution. Volodin and Dzhungurova (2000) are credited with extending the BS distribution for the first time. The generalized BS distribution was then introduced by Díaz-García and Leiva (2005); see Leiva et al. (2008) and Sanhueza et al. (2008). A three-parameter extension to the BS distribution was proposed by Owen (2006). Based on skew-elliptical models, Vilca and Leiva (2006) developed a BS distribution. Balakrishnan et al. (2009) used the expectation and maximization algorithm in estimating the parameters of the BS distribution and extended it using a scale-mixture of normal distributions. Based on the slash-elliptic model, Gómez et al. (2009) extended the BS distribution. An extended length-biased version of the BS distribution is provided by Leiva et al. (2009).

A truncated version of the BS distribution was examined by Ahmed et al. (2010). Mixture models based on the BS distribution were presented by Kotz et al. (2010). The epsilon-skew BS distribution has been developed by Vilca et al. (2010) and Castillo et al. (2011). Kundu et al. (2010) introduced the bivariate BS distribution and studied some of its properties and characteristics. The multivariate generalized BS distribution has been introduced, replacing the normal kernel by an elliptically symmetric kernel, by Kundu et al. (2013). The BS mixture distributions were considered in Balakrishnan et al. (2011). The beta-BS distribution has been defined by Cordeiro and Lemonte (2011). A shifted BS distribution was utilized by Leiva et al. (2011) to model wind energy flux.

The Kumaraswamy BS distribution was described by Saulo et al. (2012). Based on a non-homogeneous Poisson process, Fierro et al. (2013) generated the BS distribution. The gamma BS distribution was first introduced by Cordeiro et al. (2013). The Marshall–Olkin–BS distribution was studied by Lemonte (2013b). The exponentiated generalized BS distribution was proposed by Cordeiro and Lemonte (2014), whereas the zero-adjusted BS distribution was introduced by Leiva et al.

(2016). A generalization of BS distribution is done by Chacko et al. (2015). An exhaustive review of BS works can be seen in Balakrishnan and Kundu (2019). Using the skew-Laplace BS distribution, Naderi et al. (2020) demonstrated the modeling of finite mixtures. Benkhelifa (2021) introduced a new extension of the BS distribution based on the Weibull-G family of distributions.

1.3.3 Stress-Strength Reliability

There has been a long history of SS reliability, beginning with the pioneering work of Birnbaum (1956) and Birnbaum and McCarty (1958). Church and Harris (1970) are credited with introducing the term SS reliability. Kotz and Pensky (2003) provided an excellent overview of the various SS models up to 2001. Several authors have published articles on SS models recently. From a reliability perspective, Gupta and Brown (2001) investigated the skew-normal distribution and obtained the strength-stress reliability. See Raqab and Kundu (2005), Kundu and Gupta (2005, 2006), Kundu and Raqab (2009), and Sharma et al. (2015) for further details.

Sometimes the actual SS reliability of the system cannot be evaluated. It is easy to compute R, if the stress and strength are assumed to or fitted to have some well-known statistical distribution. At the same time, if the more fitted probability distributions have more parameters, then the problem becomes complicated. In such situations, one has to estimate the SS reliability, if the values of parameters are not available. SS reliability estimation is very important to investigate the level of strength and level of stress for required reliability. The estimation of SS reliability is more complicated for single-component and multi-component systems. A substantial amount of literature exists regarding the problems associated with the estimation of reliability for single-component SS models. SS reliability analysis using various statistical distributions are available in the literature.

Researchers discuss in detail the estimation of R using various statistical distributions. Using a bivariate Pareto model, Hanagal (1997) derived the maximum likelihood estimator (MLE) of the SS parameter R. A finite mixture of inverse Gaussian distributions was used by Akman et al. (1999) to study reliability estimation. A finite mixture of lognormal components is used by AI-Hussaini and Sultan (2001) to study the estimation of $R = P(Y < X)$. The exponential strength and stress random variables were taken into account by Krishnamoorthy et al. (2007). The estimation of R for the three-parameter generalized exponential distribution was investigated by Raqaab et al. (2008). Lai and Balakrishnan (2009) estimated R in models with correlated stress and strength. A study by Al-Mutairi et al. (2013) examined R estimates based on Lindley distributions. The estimation of reliability $R = P(Y < X)$ where X and Y are independent random variables that follow the Kumaraswamy distribution with varying parameters was discussed by Nadar et al. (2014). Different estimators of the parameter R were produced by Nadar and Kizilaslan (2014) in the context of the Kumaraswamy model with upper record values. Ghitany et al. (2015) discussed the reliability of SS systems based on power Lindley distributions. For a transmuted Rayleigh distribution, Dey et al. (2017) calculated the SS reliability R.

Using progressive first-failure censoring, Krishna et al. (2019) obtain R based on the inverse Weibull distribution. Rao et al. (2019) considered the estimation of stress-strength reliability based on two independent exponential inverse Rayleigh distributions that share a common scale parameter but have different shape parameters. The SS reliability estimation of single and multi-component systems has been studied by Jose et al. (2019) and Xavier and Jose (2021a) based on generalizations of half logistic distributions. On the basis of discrete phase-type distributions, Jose et al. (2022) estimated SS reliability for single and multi-component systems. Deepthi and Chacko (2020b) discussed single-component SS reliability and multi-component SS reliability estimation using the three-parameter generalized Lindley distribution. Alamri et al. (2021) estimated the SS reliability when stress and strength both follow the Rayleigh-half-normal distribution. Assuming that the strength components are distributed independently and identically as power-transformed half-logistic distributions subject to common stress, which is assumed to be independent of either the Weibull distribution or the PHL distribution, Xavier and Jose (2021b) investigated the reliability of the multicomponent stress–strength model. Varghese and Chacko (2022) examined SS reliability using the Akash distribution. Sonker et al. (2023) established stress–strength reliability models for power-Muth distribution.

1.3.4 Accelerated Life Testing

ALT was introduced by Chernoff (1962) and Bessler et al. (1962). A variety of methods can be used for accelerated testing to shorten the life of products or accelerate their degradation. During such tests, it is necessary to obtain data quickly that can be modeled and analyzed. This will enable us to generate the desired information about the product's lifetime and performance under normal conditions. Performing such tests saves a great deal of time and money. There are many ways in which accelerated tests can apply stress loading. Stress loading can be a constant, cyclic, step, or progressive. A discussion of these types of ALT is provided by Nelson (1990).

During CSALT, each unit of the test is monitored until it fails, maintaining constant levels of all stress factors. It has been found that accelerated test models for constant stress are better developed and more reliable for certain materials and products. There are several examples of constant stresses, including temperature, voltage, and current. A comprehensive review of CSALT models can be found in the works of Meeker and Escobar (1998), Escobar and Meeker (2006), Aly and Bleed (2013), and Abdel-Ghaly et al. (2016a). The method proposed by Kim and Bai (2002) for estimating the lifetime distribution for constant stress ALTs uses a mixture of two distributions to describe failure modes. To determine the lifetime of vacuum fluorescent displays, Zhang and Wang (2009) performed four CSALTs with the cathode temperature increase and assumed that the lifetime distribution was lognormal. While Bhattacharyya and Soejoeti (1981) applied the least square method under CSALT to Weibull, exponential, and gamma distributions, Bhattacharyya and Fries (1982) applied it to inverse Gaussian distributions. Using an exponentiated distribution family, Abdel-Ghaly et al. (2016b) examined different estimation methods for CSALT. For CSALT, different estimation methods under the exponentiated power Lindley distribution were discussed by Kumar et al. (2022).

The PSALT procedure involves continuously increasing stress levels on a specimen. In the study of metal fatigue, this test is used to determine the endurance limit of metal. It is likely to be difficult to control the accuracy of the PSALT. It is common for some products to undergo cyclic stress loading when they are in use. As an example, insulation under AC voltage experiences sinusoidal stress. This type of product is subjected to cyclic stress testing by repeatedly subjecting it to the same stress pattern at high levels of stress. Yin and Sheng (1987) examined the MLE of exponential failure times under PSALT. Using the Weibull distribution, Abdel-Hamid and Al-Hussaini (2011) described PSALT under progressive censoring. Using type II progressively censored data from a half-logistic distribution under PSALT, Al-Hussaini et al. (2015) calculated one-sample Bayesian prediction intervals. For the extension of the exponential distribution, Mohie El-Din et al. (2017) investigated both classical and Bayesian inference on progressively type-II censored PSALT. An inference of PSALT was discussed by Kumar Mahto et al. (2020) for the Logistic exponential distribution under progressive type-II censoring.

1.3.5 Step-Stress Accelerated Life Testing

In recent years, SSALT has become one of the most frequently discussed ALTs. This is because the level of stress on each unit increases step by step at predetermined intervals or upon a fixed number of failures. There has been extensive research on SSALTs with exponential lifetime distributions under CEM. Balakrishnan (2009) has written an excellent review article on this topic for the benefit of interested readers. Many authors have examined the optimality of an SSALT in the presence of exponential CEM, such as Miller and Nelson (1983), Bai et al.(1989), Wu et al.(2008), and Kateri et al. (2011) for different censoring methods.

Xiong (1998) discusses the inferences that can be drawn from any sample size considering an exponential lifetime distribution at constant stress and a CEM for the two-step ALT. Weibull CE model properties were examined using SSALT data by Komori (2006). When competing risk factors are independently and exponentially distributed, Balakrishnan and Han (2008) and Han and Balakrishnan (2010) investigated the SSALT under type-II and type-I censoring schemes respectively. The Weibull PH model employed in SSALT was subjected to Bayesian analysis and compared with Weibull CEM by Sha and Pan (2014). Hamada (2015) suggested and explored a generic Bayesian approach to SSALT planning.

It has been proposed by Han and Kundu (2014) that the problem of estimating point and interval estimates may be solved when the distributions of the different risk factors are s-independent Generalized Exponential distributions. El-Din et al. (2016) investigated parametric inference on SSALT for the extension of the exponential distribution with progressive type-II censoring. Chandra et al. (2017) have investigated the optimal quadratic SSALT plan for Weibull distributions with type I censoring. Hakamipour and Rezaei (2017) explored the optimization of simple SSALT using type I censoring for Frechet data. Using a simple SSALT and type II censoring, Basak and Balakrishnan (2018) predicted the censored exponential lifetimes. For a simple SSALT CEM with censored exponential data, Zhu et al. (2020) described exact likelihood-ratio tests. Under the CEM assumption, Samanta et al. (2020) develop a step-stress model with exponential distribution and evaluates the related conclusions based on Type II hybrid stress changing time. Kannan and Kundu (2020) proposed a generalized cumulative risk model for simple SSALT and developed this model on the premise that the underlying population comprised both 'cured' individuals and susceptible individuals. Pal et al. (2021) introduced the failure rate-based simple step-stress model for the Lehmann family of distributions.

Nonparametric methods do not assume the existence of a specific lifetime distribution. By utilizing this distribution-free strategy, we can mitigate the significant error in extrapolating SSALT results when there is a bias in the evaluation of the potential lifetime models or when the models do not provide a good representation of the failure mechanism. For determining the upper confidence bounds of the cumulative failure probability of a product, Hu et al. (2012) suggested a nonparametric PHM. Other research on this subject is found in [Schmoyer (1991) and Pascual and Montepiedra (2003)].

In-depth research has been done on random effects in lifetime trials. When the group impact is statistically significant, León et al. (2007) established a Bayesian approach to conclude ALT data. They compare fixed and random group effect models and demonstrate that the latter offers more precise predictions and estimates. To account for the random group effect in SSALT, Seo and Pan (2017) suggested a generalized linear mixed-effect model. The results from two estimating techniques—adaptive Gaussian quadrature and integrated nested Laplace Approximation—are analyzed. Wang (2020) took into account the Weibull distribution-based data analysis of SSALT data with random group effects.

1.4 Motivation of the present work

Reliability engineering and statistical modeling can benefit from introducing new generalized distributions because they can provide customized solutions for specific challenges, improve model accuracy, foster innovation, handle complex system behaviors, deepen understanding of statistical theory, facilitate interdisciplinary applications, and adapt to emerging data types. Even though existing generalized distributions are valuable, new distributions must be developed to address changing needs and technological advances. The fitness of distributions to the given data is important to draw valid and accurate probability computations. This enables us to accurately model a wide range of real-world situations and fosters cross-disciplinary collaboration. A search for new distributions for modeling lifetime data is essential in this context.

When one considers a parallel system where each of the components has DUS-transformed distributions for its lifetime, we should investigate the distributional properties. Moreover, we have to investigate the distributional properties of the parallel system when components are distributed as DUS transformations of baseline distributions like exponential, Weibull, and Lomax distributions. There are several other distributions that can serve as baseline distributions. After investigating the flexibility of the new distributions in terms of simulation, fitness, estimation, etc. using exponential, Weibull, and Lomax distributions, we can go for other distributions. Generalized exponential distribution (Gupta and Kundu (1999)) was widely accepted by researchers since it was applicable to parallel systems in which components are exponentially distributed. But using an exponential distribution for the lifetime of a component is limited to the case of random failures. But what would be the behavior if we use any other distribution with a non-monotonic failure rate? Nowadays, distributions using the DUS transformation receive high attention since this transformation does not add any more parameters but shows better fitness than the baseline distribution. We have to investigate the power generalization of distribution using the DUS transformation to describe effectiveness, behavior, etc. An attempt towards the power generalization of DUS transformation has to be explored more. A variety of generalizations of BS distributions are available in the reliability literature. A detailed study, especially in the inference part, also has to be explored more.

Mixture distributions are useful when a new component switches on for the first time. They may fail at the same instant of starting operation, or they may fail due to overvoltage, jerking, or any such shocks, or they may fail due to the degradation of the component. Failure due to random shocks is modelled using an exponential distribution. Failure due to degradation can be modelled using any other distribution with a non-monotonic failure rate. In statistical modeling and analysis, for reliability and survival analysis studies, the introduction of a mixture distribution based on exponential and gamma distributions can be seen as extremely useful. In this way, complex data patterns can be captured that cannot be adequately captured by a single distribution. The main purpose of mixture models is to enhance the understanding and description of real-world phenomena by combining several different distributions. A detailed study of mixture distributions has to be carried out to examine failure rate behavior and its inference. The determination of stress-strength reliability has to be addressed in mixture distributions. Estimation of stress-strength reliability is also a research problem when using mixture models. After investigating some mixture distributions and their usefulness in stress-strength analysis, we can investigate the remaining mixtures as per need.

Step-stress accelerated life testing (SSALT) with Type II censoring is a method for assessing the reliability and durability of a product or system while minimizing the amount of testing time and resources required. Using Type II censoring, a product's life is estimated based on how many units fail, and the information gathered is used to determine the product's lifetime. Through SSALT, the units are subjected to progressively higher levels of stress over time or usage, causing the products to age more rapidly. Manufacturers can make informed decisions about product design improvements, warranty policies, and maintenance schedules by designing and analyzing SSALT experiments with Type II censoring to produce robust and reliable products at an affordable cost.

1.5 Objectives of the Study

1. To study the increasing failure rate, decreasing failure rate, bathtub-shaped failure rate, and upside-down bathtub-shaped failure rate distributions and their properties and applications for modeling lifetime data.

2. To study the role of bathtub-shaped failure rate distributions in system engineering and other scientific area and propose new distributions.

3. To study existing step-stress accelerated life testing (SSALT) models, develop an SSALT model, and estimate its model parameters.

4. To study the properties of Birnbaum-Saunders distributions and their generalizations on reliability theory.

5. To study on the stress-strength reliability models and its inferential procedures.

1.6 Outline of the Present Study

A total of seven chapters are included in the thesis. A new generalization of the DUS transformation, the PGDUS transformation, is presented with applications to exponential, Weibull, and Lomax distributions. A new BFR distribution called exponential-gamma $(3, \theta)$ is studied. Further, SS reliability is also calculated for this distribution. Generalization of the BS distribution called ν-BS distribution is then provided. A simple SSALT analysis of Type-II Gumbel distribution under Type-II censoring is given.

The chapters of the thesis are arranged in the following manner. *Chapter 1* provides an overview of the basic concepts and definitions used throughout this thesis. Also, an extensive literature review is given. A comprehensive review study of the IFR, DFR, BFR, UBFR, BS distribution, stress-strength reliability model distributions, and SSALT analysis was conducted to achieve the results of this research work.

Chapter 2 introduces a new transformation called the power generalized DUS transformation and proposes new distributions with exponential, Weibull, and Lomax distributions as baseline distributions. Several mathematical properties are examined, including moments, MGFs, CFs, quantile functions, order statistics, etc. The maximum likelihood approach to parameter estimation is discussed. Based on several real data sets, the proposed distributions are compared with some of the other failure rate lifetime distributions. It has been found that the new distributions fit the data better than the well-known distributions.

Chapter 3 examines in detail a new BFR distribution called the exponential-gamma $(3, \theta)$ distribution. An investigation is conducted into the shapes of the PDF and the failure rate. Various properties are discussed, including moments, MGF, CF, the quantile function, and entropy. Distributions for the minimum and maximum are discovered. In order to estimate the parameters of the distribution, the maximum likelihood method is utilized. Through the use of a simulation study, biases and mean squared errors are analyzed for maximum likelihood estimators (MLEs). A comparison between the proposed lifetime distribution and other lifetime distributions is conducted based on real data sets.

In *Chapter 4* the generalization of the BS distribution, called the ν-Birnbaum Saunders distribution, is discussed. A number of intriguing and relevant characteristics are investigated in depth. The maximum likelihood principle is employed to estimate the parameters of the univariate ν-BS distribution. To obtain interval estimates, we use asymptotic confidence intervals. Both estimation methodologies have been thoroughly explored in an extensive simulation study. Based on these estimators, the probability coverage of confidence intervals has been evaluated. Real-life applications are provided with three different datasets and compared with the univariate BS distribution.

When a manufacturer has knowledge of the mechanical reliability of the design through the stress-strength model before production, they can significantly reduce their production costs. A system's longevity is determined by its inherent strength and external stresses. A discussion of the stress-strength reliability of the exponential-gamma $(3, \theta)$ distribution is presented in *Chapter 5*. An assessment of the reliability estimation of the single-component model is provided. A simulation study is used to demonstrate how well the MLEs perform. A data application is presented using real data sets to demonstrate how the distribution performs in real-life situations.

In *Chapter 6*, a simple SSALT analysis is provided incorporating Type-II censoring. Here, a flexible failure rate-based approach to Type II Gumbel distribution for SSALT analysis is considered. The baseline distribution of experimental units at each stress level follows the Type II Gumbel distribution. The MLE for the model parameters is derived.

In *Chapter 7*, a conclusion of the thesis is presented, as well as recommendations for future research. A list of references is included at the end of the thesis.