CHAPTER 5

Inference for $\mathbf{R = P[X > Y]}$ based on the Exponential-Gamma $(\mathbf{3}, \lambda)$ Distribution

5.1 Introducion

Stress-strength (SS) reliability analysis is an important area of reliability analysis. Strength can be considered as a random variable. In light of the uncertainty in the operating environment of the unit, the stress applied to it should also be considered as a random variable. Let X represent a unit's strength, and Y represent the random stress that the operational environment imposes on the unit. $R = P(X > Y)$ is SS reliability (R).

It is easy to compute R if the stress and strength are assumed to or fitted to have some well-known statistical distribution. At the same time, if the fitted probability distributions have more parameters, then the problem becomes complicated. In such situations, one has to estimate SS reliability if the values of parameters are not available. Estimating the reliability of SS models is essential to determining strength and stress levels. The estimation of SS reliability is more complicated for single-component and multi-component systems. The problem of estimating reliability for single-component SS models is well documented in the literature.

A variety of censoring schemes have been employed in the literature to analyze

SS reliability. Based on the Gumbel copula under the type-I progressively hybrid censoring scheme, Bai et al. (2018) assessed the reliability of the multi-component SS model. Abravesh et al. (2019) assessed SS reliability with classical and Bayesian estimation methods based on type-II censored Pareto distributions. Byrnes et al. (2019) used progressively first failure-censored samples to estimate R for the Burr type XII distribution. Under progressive type II censoring, Zhang et al. (2019) examined the reliability of the generalized Rayleigh distribution. The inference of multicomponent SS reliability under progressive Type II censoring is presented by Jha et al. (2020), in which stress and strength variables have common unit Gompertz distributions. Karimi Ezmareh and Yari (2022) studied the inference of SS reliability for the Gompertz distribution using a type II censoring scheme.

The exponential-gamma $(3, \lambda)$ distribution studied in Chapter 3, which has a bathtub-shaped failure rate function, is used to analyze SS reliability.In this chapter, the exponential-gamma distribution $(3, \lambda)$ is denoted by $EGD(3, \lambda)$ or simply EGD.Specifically, $EGD(3, \lambda)$ has the PDF

$$f(x) = \frac{\lambda^2}{1+\lambda}\left(1 + \frac{\lambda}{2}x^2\right)e^{-\lambda x}, x > 0, \lambda > 0. \tag{5.1.1}$$

It should be noted that $EGD(3, \lambda)$ is a mixture of exponential distribution with a scale parameter of λ and gamma distribution with a shape parameter of 3 and a scale parameter of λ with mixing proportion $\frac{\lambda}{1+\lambda}$. It has been relatively unexplored whether SS reliability can be estimated when stress and strength vary independently following an $EGD(3, \lambda)$ distribution. This motivates the estimation of stress-strength reliability using $EGD(3, \lambda)$.

Consider two independent random variables X and Y from the $EGD(3, \lambda)$ with different parameters λ_1 and λ_2. This chapter focuses on the estimation of the parameter $R = P(X > Y)$ while stress and strength have $EGD(3, \lambda)$ distribution under type-II censoring. Typically, the problem of estimating R arises when dealing with the reliability of a component of strength X subjected to a load or stress Y. The component will fail if the stress exceeds its threshold level. As a result, R can be viewed as a measure of reliability.

The type II censoring method is briefly explained. Suppose that $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_m$ are independent random samples drawn from X and Y random variables, respectively. Consider the ordered statistics of these samples to be $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$ and $y_{(1)}, y_{(2)}, \ldots, y_{(m)}$. The x_i's and y_i's are collected until failure occurs at r_1 and r_2 (where r_1 is less than or equal to n and r_2 is less than or equal to m).

Our goal in this chapter is to estimate the SS reliability when both stress and strength follow EGD with different parameters λ_1 and λ_2 under the type II censoring scheme. Section 5.2 considers the SS reliability of $EGD(3, \lambda)$. In section 5.3, the MLE of R using type-II censoring, the asymptotic distribution, and the CI for the MLE of R are obtained. An extensive simulation study is presented in section 5.4. Section 5.5 presents the results of the analysis of real data. In the final section, a summary is given.

5.2 Stress-Strength Reliability of $EGD(3, \lambda)$ Distribution

In this section, SS reliability is estimated using the EGD distribution. The general mathematical expression of SS reliability for the independent random variables X and Y is given by

$$R = \int_{-\infty}^{\infty} f_X(x)\, F_Y(x)\, dx,$$

where $f_X(x)$ and $F_Y(x)$ are the marginal PDF of X and marginal CDF of Y, respectively.

Consider X and Y as independent random variables having the EGD distribution with parameters λ_1 and λ_2, respectively. Suppose $X \sim EGD(3, \lambda_1)$ and $Y \sim EGD(3, \lambda_2)$. Then, SS reliability is

$$R = \int_0^{\infty} \frac{\lambda_1^2}{(1+\lambda_1)} \left(1 + \frac{\lambda_1}{2}x^2\right) e^{-\lambda_1 x} \left[1 - \frac{(\lambda_2(x(\lambda_2 x + 2) + 2) + 2)e^{-\lambda_2 x}}{2(1+\lambda_2)}\right] dx$$

$$= \tfrac{\lambda_1^2}{2(1+\lambda_1)(1+\lambda_2)} \int_0^{\infty} \left(1 + \tfrac{\lambda_1 x^2}{2}\right) e^{-\lambda_1 x} [2(1+\lambda_2) - (\lambda_2^2 x^2 + 2x\lambda_2 + 2\lambda_2 + 2)e^{-\lambda_2 x}] dx$$

$$= \frac{\lambda_1^2}{(1+\lambda_1)} \int_0^{\infty} \left(1 + \frac{\lambda_1}{2}x^2\right) e^{-\lambda_1 x} dx$$

$$- \tfrac{\lambda_1^2}{2(1+\lambda_1)(1+\lambda_2)} \int_0^{\infty} \left(1 + \tfrac{\lambda_1}{2}x^2\right) e^{-(\lambda_1+\lambda_2)x} (\lambda_2^2 x^2 + 2x\lambda_2 + 2\lambda_2 + 2) dx$$

$$=1-\left[\frac{\lambda_1^2(\lambda_2^2+\lambda_1\lambda_2+\lambda_1)}{(1+\lambda_1)(1+\lambda_2)(\lambda_1+\lambda_2)^3}+\frac{\lambda_1^2\lambda_2}{(1+\lambda_1)(1+\lambda_2)(\lambda_1+\lambda_2)^2}+\frac{\lambda_1^2}{(1+\lambda_1)(\lambda_1+\lambda_2)}\right.$$

$$\left.+\frac{6\lambda_1^3\lambda_2^2}{(1+\lambda_1)(1+\lambda_2)(\lambda_1+\lambda_2)^5}+\frac{3\lambda_1^3\lambda_2}{(1+\lambda_1)(1+\lambda_2)(\lambda_1+\lambda_2)^4}\right]$$

$$=\frac{\lambda_2(10\lambda_1^2\lambda_2^2+5\lambda_1\lambda_2^3+\lambda_2^4+12\lambda_1^2\lambda_2^3+6\lambda_1\lambda_2^4)}{(1+\lambda_1)(1+\lambda_2)(\lambda_1+\lambda_2)^5}$$

$$+\frac{\lambda_2(3\lambda_1^4\lambda_2+10\lambda_1^3\lambda_2^2+\lambda_2^5+\lambda_1^5\lambda_2+4\lambda_1^4\lambda_2^2+6\lambda_1^3\lambda_2^3+4\lambda_1^2\lambda_2^4+\lambda_1\lambda_2^5)}{(1+\lambda_1)(1+\lambda_2)(\lambda_1+\lambda_2)^5} \tag{5.2.1}$$

This expression evaluates if the values of parameters are available. But in practice, it is not available. Hence, one has to estimate the parameters to determine reliability.

5.3 Maximum Likelihood Estimator of R

Let us suppose that $X_{(1)}, X_{(2)}, \ldots, X_{(r_1)}$ is a type II censored sample from $EGD(3, \lambda_1)$ and $Y_{(1)}, Y_{(2)}, \ldots, Y_{(r_2)}$ is a type II censored sample from $EGD(3, \lambda_2)$. The two samples are assumed to be independent. The joint likelihood function is

$$L=\frac{n!\,m!}{(n-r_1)!(m-r_2)!}\frac{\lambda_1^{2r_1}}{(1+\lambda_1)^{r_1}}e^{-\lambda_1\sum_{k=1}^{r_1}x_{(k)}}\frac{\lambda_2^{2r_2}}{(1+\lambda_2)^{r_2}}e^{-\lambda_2\sum_{l=1}^{r_2}y_{(l)}}\left(\frac{1}{2(1+\lambda_1)}\right)^{n-r_1}\left(\frac{1}{2(1+\lambda_2)}\right)^{m-r_2}$$

$$\prod_{l=1}^{r_2}\left(1+\frac{\lambda_2}{2}y_{(l)}^2\right)\left[(\lambda_2(y_{(r_2)}(\lambda_2 y_{(r_2)}+2)+2)+2)e^{-\lambda_2 y_{(r_2)}}\right]^{m-r_2}$$

$$\prod_{k=1}^{r_1}\left(1+\frac{\lambda_1}{2}x_{(k)}^2\right)\left[(\lambda_1(x_{(r_1)}(\lambda_1 x_{(r_1)}+2)+2)+2)e^{-\lambda_1 x_{(r_1)}}\right]^{n-r_1}. \tag{5.3.1}$$

The log-likelihood associated with the above equation is given by

$$\log L=\log(n!)+\log(m!)-\log((n-r_1)!)-\log((m-r_2)!)+2r_1\log(\lambda_1)+2r_2\log(\lambda_2)$$

$$-r_1\log(1+\lambda_1)-r_2\log(1+\lambda_2)-\lambda_2\sum_{l=1}^{r_2}y_{(l)}-(n-r_1)\log(2(1+\lambda_1))$$

$$-(m-r_2)\log(2(1+\lambda_2))+\sum_{k=1}^{r_1}\log\left(1+\frac{\lambda_1}{2}x_{(k)}^2\right)+\sum_{l=1}^{r_2}\log\left(1+\frac{\lambda_2}{2}y_{(l)}^2\right)$$

$$-\lambda_1\sum_{k=1}^{r_1}x_{(k)}+(n-r_1)\log(\lambda_1(x_{(r_1)}(\lambda_1x_{(r_1)}+2)+2)+2)-(n-r_1)\lambda_1x_{(r_1)}$$

$$+(m-r_2)\log(\lambda_2(y_{(r_2)}(\lambda_2y_{(r_2)}+2)+2)+2)-(m-r_2)\lambda_2\,y_{(r_2)}$$

The first derivative of the above log-likelihood equation with respect to the unknown parameters λ_1 and λ_2 are respectively given by

$$\frac{\partial\log L}{\partial\lambda_1}=\frac{2r_1}{\lambda_1}-\frac{r_1}{1+\lambda_1}-\sum_{k=1}^{r_1}x_{(k)}-\frac{(n-r_1)}{1+\lambda_1}-(n-r_1)x_{(r_1)}+\sum_{k=1}^{r_1}\frac{x_{(k)}^2}{(2+\lambda_1x_{(k)}^2)}$$

$$+2(n-r_1)\frac{(1+x_{(r_1)}+\lambda_1x_{(r_1)}^2)}{(\lambda_1(x_{(r_1)}(\lambda_1x_{(r_1)}+2)+2)+2)}$$

$$\frac{\partial\log L}{\partial\lambda_2}=\frac{2r_2}{\lambda_2}-\frac{r_2}{1+\lambda_2}-\sum_{l=1}^{r_2}y_{(l)}-\frac{(m-r_2)}{(1+\lambda_2)}-(m-r_2)y_{(r_2)}+\sum_{l=1}^{r_2}\frac{y_{(l)}^2}{(2+\lambda_2y_{(l)}^2)}$$

$$+2(m-r_2)\frac{(1+y_{(r_2)}+\lambda_2y_{(r_2)}^2)}{(\lambda_2(x_{(r_2)}(\lambda_2y_{(r_2)}+2)+2)+2)}.$$

The second derivative of the above log-likelihood equation with respect to the unknown parameters λ_1 and λ_2 are respectively given by

$$\frac{\partial^2\log L}{\partial\lambda_1^2}=\frac{n}{(1+\lambda_1)^2}-\frac{2r_1}{\lambda_1^2}-\sum_{k=1}^{r_1}\frac{x_{(k)}^4}{(2+\lambda_1x_{(k)}^2)^2}-2(n-r_1)\frac{(x_{(r_1)}(\lambda_1x_{(r_1)}+2)(\lambda_1x_{(r_1)}^2+2)+2)}{(\lambda_1(x_{(r_1)}(\lambda_1x_{(r_1)}+2)+2)+2)^2}.$$

$$\frac{\partial^2 \log L}{\partial \lambda_2^2} = \frac{m}{(1+\lambda_2)^2} - \frac{2r_2}{\lambda_2^2} - \sum_{l=1}^{r_2} \frac{y_{(l)}^4}{(2+\lambda_2 y_{(l)}^2)^2} - 2(m-r_2)\frac{(y_{(r_2)}(\lambda_2 y_{(r_2)}+2)(\lambda_2 y_{(r_2)}^2+2)+2)}{(\lambda_2(y_{(r_2)}(\lambda_2 y_{(r_2)}+2)+2)+2)^2}.$$

Using Eq.(5.2.1), MLE of SS reliability, $\hat{R}_{ML}$, can be calculated as follows:

$$\hat{R}_{ML} = \frac{\hat{\lambda}_2(10\hat{\lambda}_1^2\hat{\lambda}_2^2+5\hat{\lambda}_1\hat{\lambda}_2^3+\hat{\lambda}_2^4+12\hat{\lambda}_1^2\hat{\lambda}_2^3+6\hat{\lambda}_1\hat{\lambda}_2^4+3\hat{\lambda}_1^4\hat{\lambda}_2+10\hat{\lambda}_1^3\hat{\lambda}_2^2+\hat{\lambda}_2^5+\hat{\lambda}_1^5\hat{\lambda}_2+4\hat{\lambda}_1^4\hat{\lambda}_2^2+6\hat{\lambda}_1^3\hat{\lambda}_2^3+4\hat{\lambda}_1^2\hat{\lambda}_2^4+\hat{\lambda}_1\hat{\lambda}_2^5)}{(1+\hat{\lambda}_1)(1+\hat{\lambda}_2)(\hat{\lambda}_1+\hat{\lambda}_2)^5}. \tag{5.3.2}$$

Asymptotic Distribution and Confidence Intervals

The asymptotic distribution and confidence interval (CI) for the MLE of R are given in this section. Let us represent the Fisher information matrix of $\lambda = (\lambda_1, \lambda_2)$ as $I(\lambda)$. In order to obtain the asymptotic variance of the MLE of R, $\hat{R}_{ML}$, use $I(\lambda)$ in Eq.(5.3.2), where

$$I(\lambda) = E\begin{bmatrix} -\frac{\partial^2 \log L}{\partial \lambda_1^2} & -\frac{\partial^2 \log L}{\partial \lambda_1 \partial \lambda_2} \\ -\frac{\partial^2 \log L}{\partial \lambda_2 \partial \lambda_1} & -\frac{\partial^2 \log L}{\partial \lambda_2^2} \end{bmatrix}.$$

The asymptotic normality of R is obtained by using the following definition

$$d(\lambda) = \left(\frac{\partial R}{\partial \lambda_1}, \frac{\partial R}{\partial \lambda_2}\right)' = (d_1, d_2)'$$

where

$$\frac{\partial R}{\partial \lambda_1} = -\frac{\lambda_1\lambda_2^2(\lambda_1^5+(4\lambda_2+6)\lambda_1^4+(+\lambda_2^2+20\lambda_2+3)\lambda_1^3+(4\lambda_2^3+24\lambda_2^2+48\lambda_2)\lambda_1^2+(\lambda_2^4+12\lambda_2^3+21\lambda_2^2+30\lambda_2)\lambda_1+2\lambda_2^4+6\lambda_2^3)}{(1+\lambda_1)^2(1+\lambda_2)(\lambda_1+\lambda_2)^6}$$

and

$$\frac{\partial R}{\partial \lambda_2} = \frac{\lambda_1^2\lambda_2(\lambda_2^5+2(2\lambda_1+3)\lambda_2^4+(6\lambda_1^2+20\lambda_1)\lambda_2^3+4\lambda_1(\lambda_1^2+6\lambda_1+12)\lambda_2^2+\lambda_1(\lambda_1^3+12\lambda_1^2+21\lambda_1+30)\lambda_2+2\lambda_1^4+6\lambda_1^3}{(1+\lambda_1)^2(1+\lambda_2)(\lambda_1+\lambda_2)^6}$$

As a result, the asymptotic distribution of $\hat{R}_{ML}$ can be represented as

$$\sqrt{n+m}(\hat{R}_{ML} - R) \rightarrow^{d} N(0, d'(\lambda)\, I^{-1}(\lambda)\, d(\lambda)).$$

We obtain the asymptotic variance of $\hat{R}_{ML}$ as follows:

$$AV(\hat{R}_{ML}) = \frac{1}{n+m} d'(\lambda)\, I^{-1}(\lambda)\, d(\lambda)$$

$$= V(\hat{\lambda_1})d_1^2 + V(\hat{\lambda_1})d_2^2 + 2d_1 d_2(\hat{\lambda_1}\hat{\lambda_1}).$$

Asymptotic $100(1-\omega)\%$ CI for R can be obtained as

$$\hat{R}_{ML} \pm Z_{\omega/2}\sqrt{AV(\hat{R}_{ML})}$$

where $Z_{\omega/2}$ is the upper $\omega/2$ quantile of the standard normal distribution. To assess the efficiency of the estimators, a simulation study is carried out and given in next section.

5.4 Simulation Study

This section presents some results related to the performance of estimators of R using the Newton-Raphson method. For this purpose, 1000 samples are generated using independent $EGD(3, \lambda_1)$ and $EGD(3, \lambda_2)$ distributions for various sample sizes under type II censoring scheme. The parameter values, (λ_1, λ_2), used in this study were (0.5, 1.5), (1, 1.5), and (1.5, 0.5). Corresponding to these parameter values, R values are 0.8391, 0.6405, and 0.1609, respectively.

Tables 5.1- 5.3 provided estimates of R based on the MLE method along with average biases, mean square errors (MSEs), and 95% CIs. From these simulation results, biases and MSEs decrease with increasing sample size (n, m).

Table 5.1: MLE, average(Avg) bias, and MSEs of different estimators of R when $\lambda_1 = 0.5$ and $\lambda_2 = 1.5$.

(n,m)	$(\mathbf{r_1}, \mathbf{r_2})$	**Avg Bias**	**MSEs**	**95% CI**	**Estimates**
(15,15)	(15,15)	0.0124	0.0080	(0.7361, 0.9523)	$\hat{\lambda}_1$=0.5124
		0.0878	0.1124		$\hat{\lambda}_2$=1.5878
	(14,14)	0.0193	0.0091	(0.6832, 0.9927)	$\hat{\lambda}_1$=0.5193
		0.0589	0.1044		$\hat{\lambda}_2$=1.5589
	(12,12)	0.0215	0.0105	(0.7259, 0.9563)	$\hat{\lambda}_1$=0.5215
		0.0940	0.1412		$\hat{\lambda}_2$=1.5940
(25,25)	(25,25)	0.0113	0.0046	(0.7499, 0.9290)	$\hat{\lambda}_1$=0.5113
		0.0430	0.0557		$\hat{\lambda}_2$=1.5430
	(23,23)	0.0114	0.0046	(0.7648, 0.9185)	$\hat{\lambda}_1$=0.5114
		0.0621	0.0643		$\hat{\lambda}_2$=1.5621
	(21,21)	0.0135	0.0057	(0.7713, 0.9072)	$\hat{\lambda}_1$=0.5135
		0.0493	0.0642		$\hat{\lambda}_2$=1.5493
(30,30)	(30,30)	0.0088	0.0040	(0.7615, 0.9197)	$\hat{\lambda}_1$=0.5089
		0.0440	0.0454		$\hat{\lambda}_2$=1.5440
	(28,28)	0.0089	0.0041	(0.7638, 0.9123)	$\hat{\lambda}_1$=0.5099
		0.0231	0.0442		$\hat{\lambda}_2$=1.5231
	(25,25)	0.0087	0.0048	(0.7584, 0.9201)	$\hat{\lambda}_1$=0.5087
		0.0324	0.0511		$\hat{\lambda}_2$=1.5324

Table 5.2: MLE, Avg bias, and MSEs of different estimators of R when $\lambda_1 = 1$ and $\lambda_2 = 1.5$.

(n,m)	$(\mathbf{r_1}, \mathbf{r_2})$	**Avg Bias**	**MSEs**	**95% CI**	**Estimates**
(15,15)	(15,15)	0.0322	0.0382	(0.4666, 0.8290)	$\hat{\lambda}_1$=1.0322
		0.0878	0.1124		$\hat{\lambda}_2$=1.5878
	(14,14)	0.0477	0.0447	(0.3602, 0.9138)	$\hat{\lambda}_1$=1.0477
		0.0589	0.1044		$\hat{\lambda}_2$=1.5589
	(12,12)	0.0479	0.0481	(0.4690, 0.8167)	$\hat{\lambda}_1$=1.0479
		0.0885	0.1253		$\hat{\lambda}_2$=1.5885
(25,25)	(25,25)	0.0281	0.0218	(0.4877, 0.7925)	$\hat{\lambda}_1$=1.0281
		0.0430	0.0557		$\hat{\lambda}_2$=1.5430
	(23,23)	0.0280	0.0222	(0.5142, 0.7738)	$\hat{\lambda}_1$=1.0280
		0.0621	0.0643		$\hat{\lambda}_2$=1.5621
	(21,21)	0.0332	0.0279	(0.5269, 0.7526)	$\hat{\lambda}_1$=1.0332
		0.0493	0.0642		$\hat{\lambda}_2$=1.5493
(30,30)	(30,30)	0.0208	0.0173	(0.5233, 0.7624)	$\hat{\lambda}_1$=1.0208
		0.0447	0.0470		$\hat{\lambda}_2$=1.5447
	(28,28)	0.0242	0.0202	(0.5100, 0.7700)	$\hat{\lambda}_1$=1.0242
		0.0362	0.0463		$\hat{\lambda}_2$=1.5380
	(25,25)	0.0254	0.0209	(0.5195, 0.7626)	$\hat{\lambda}_1$=1.0254
		0.0434	0.0535		$\hat{\lambda}_2$=1.5434

Table 5.3: MLE, Avg bias, and MSEs of different estimators of R when $\lambda_1 = 1.5$ and $\lambda_2 = 0.5$.

(n,m)	$(\mathbf{r_1}, \mathbf{r_2})$	**Avg Bias**	**MSEs**	**95% CI**	**Estimates**
(15,15)	(15,15)	0.0618	0.0913	(0.0564, 0.2635)	$\hat{\lambda}_1$=1.5618
		0.0151	0.0080		$\hat{\lambda}_2$=0.5151
	(14,14)	0.0682	0.1229	(0.0472, 0.2696)	$\hat{\lambda}_1$=1.5682
		0.0132	0.0078		$\hat{\lambda}_2$=0.5132
	(12,12)	0.0826	0.1231	(0.0369, 0.2811)	$\hat{\lambda}_1$=1.5826
		0.0186	0.0099		$\hat{\lambda}_2$=0.5186
(25,25)	(25,25)	0.0406	0.0516	(0.0820, 0.2431)	$\hat{\lambda}_1$=1.5369
		0.0141	0.0048		$\hat{\lambda}_2$=0.5141
	(23,23)	0.0387	0.0620	(0.0881, 0.2340)	$\hat{\lambda}_1$=1.5381
		0.0111	0.0050		$\hat{\lambda}_2$=0.5111
	(21,21)	0.0464	0.0659	(0.0700, 0.2506)	$\hat{\lambda}_1$=1.5464
		0.0117	0.0056		$\hat{\lambda}_2$=0.5117
(30,30)	(30,30)	0.0369	0.0463	(0.0923, 0.2264)	$\hat{\lambda}_1$=1.5406
		0.0078	0.0035		$\hat{\lambda}_2$=0.5078
	(28,28)	0.0381	0.0478	(0.0873, 0.2304)	$\hat{\lambda}_1$=1.5387
		0.0060	0.0040		$\hat{\lambda}_2$=0.5060
	(25,25)	0.0312	0.0525	(0.0989, 0.2245)	$\hat{\lambda}_1$=1.5312
		0.0107	0.0042		$\hat{\lambda}_2$=0.5107

5.5 Applications

To check the applicability of the model we considered the dataset used by Sonker et al. (2023) which is extracted from the dataset available in Andrews and Herzberg (2012) and contains information on Kevlar pressure vessels' stress rupture life under constant pressure. With $r_1 = r_2 = 19$, Type II right censoring is performed on the complete dataset.The data are presented as follows.

X : 6121, 11604, 9711, 9106, 11026, 17568, 1921, 4921, 10861, 11214, 11608, 5956, 1337, 10205, 11745, 2322, 16179, 14110, 7501, 8666.

Y : 1942, 17568, 3629, 11362, 4006, 14496, 6068, 7886, 5905, 6473, 11895, 4012, 13670, 10396, 17092, 8108, 1051, 5445, 5817, 5620.

When the EGD model is fitted to the data, it can be seen that the model fits the data quite well. Similarly, the following data is fitted with the Lindley (LD) model, and it can be observed that the EGD model provides a better fit to the data than the LD model. Since, the EGD model has minimum CVM and KS values, and maximum p-values.

The MLE for parameters λ_1 and λ_2, Cramer-Von Mises (CVM) and the Kolmogorov-Smirnov (K-S) tests are given in Table 5.4 and 5.5. As a result, the MLE of R of EGD model is $\hat{R} = 0.5002$, and the 95% CI for R is (0.2805,0.7199).

Table 5.4: MLE, CVM, and KS goodness of fit tests for X data

Data	Estimates	CVM (p-value)	KS (p-value)
LD	0.0002	0.1883 (0.2929)	0.1836 (0.4873)
EGD	0.0003	0.1297 (0.4614)	0.1683 (0.5967)

Table 5.5: MLE, CVM, and KS goodness of fit tests for Y data

Data	Estimates	CVM (p-value)	KS (p-value)
LD	0.0002	0.1883 (0.2929)	0.1836 (0.4872)
EGD	0.0003	0.1299 (0.4607)	0.1680 (0.5989)

5.6 Summary

There have been several well-developed estimation techniques for SS models with single components that follow well-known lifetime distributions. The EGD model is found to be a better model than some existing models. In this chapter, the problem of estimating SS reliability with an EGD distribution in a single-component SS model for independent stress and strength random variables under type-II censoring is discussed in detail. The MLE of SS reliability, $\hat{R}_{ML}$, is obtained. The extensive simulation revealed that the MSE and average biases caused by estimates approach zero when sample sizes are increased. The analysis is conducted on real-life datasets and compares the EGD model with the Lindley model. The EGD model is found to be a good fit, and it can be used for SS reliability analysis.