CHAPTER 7

Conclusion and Future Directions

7.1 Conclusion

A number of methods are available in the statistical literature to propose new distributions based on baseline distributions. In statistical distribution theory, adding a parameter to a family of distribution functions is a very common practice. In the context of data analysis, adding a parameter can greatly enhance the flexibility of a class of distribution functions. The DUS transformation to any of the lifetime distributions proved to be an alternative approach to getting more flexible models without adding new parameters. As in the generalized exponential distribution, we can find the distribution of the parallel system in which we can use the DUS transformed distribution instead of the simple exponential distribution. A generalization is made by taking power to the DUS transformation, which is quite desirable since the DUS transformation of any non-monotonic failure rate model leads to new better models without increasing parameters. Three new distributions are introduced based on this generalized transformation, the PGDUS transformation, using the baseline distributions, exponential, Weibull, and Lomax. PGDUS distribution is a distribution of $max(X_1, X_2, \ldots, X_n)$, where $(X_1, X_2, \ldots, X_n)$ follows DUS-transformed distributions. The PGDUS approach is highly useful when performing reliability analyses on a parallel system whose components have DUS-transformed lifetime distributions. The new distributions are studied in detail and investigated for their properties.

The use of statistical distributions plays a significant role in solving a variety of real-life problems. The use of mixture distributions is unavoidable since, in many real-life situations, instead of using a particular model, we have to use mixture models. The importance of mixture statistical distributions in reliability analysis led us to study a statistical distribution with a bathtub-shaped failure rate function called the exponential-gamma $(3, \theta)$ distribution. The distribution is studied in detail and has several properties.

Birnbaum-Saunders distributions can be applied in a variety of contexts when fatigue failure occurs. In particular, BS distributions have been applied to failure models in random environments characterized by stationary Gaussian processes. In addition, the BS fatigue life distribution can be used to efficiently model wear-out and cumulative damage situations. The genesis of this model makes it evident that fatigue processes are best modeled by this distribution. Many different fields have used BS distributions. As an example, in the earth sciences, particularly rain precipitation; in acceptance sampling and quality control; in warranty claim prediction; and in medicine. Additionally, the BS distribution is closely related to the inverse Gaussian distribution, which makes it an ideal distribution for use in actuarial science, demography, agriculture, economics, finance, toxicology, hydrology, environmental sciences, and wind energy. Further generalizations to the univariate BS distribution are considered. ν-BS distribution is one of the generalizations of BS distribution. But the estimation procedures for the ν-BS distribution were not available in the literature. In order to examine the usefulness of the ν-BS distribution, a detailed study is required.

Among the univariate, bivariate, and multivariate cases of the ν-BS distribution, the univariate case is discussed in detail. An in-depth study is conducted on some of their properties. By applying the maximum likelihood principle, point estimates for the univariate ν-BS distribution are obtained. Asymptotic CIs are calculated using the observed information matrix to obtain interval estimates. A comprehensive simulation study was conducted to examine the validity of the model. A comparison of the $\nu-$BS distribution using three real data sets is presented to demonstrate that the ν-BS distribution consistently provides better modeling than the BS distribution.

Statistical analysis of the SS reliability of a component consists of analyzing how the strength of the component and the stresses placed upon it interact.It is pertinent to consider the stress conditions of the environment in which it operates when determining an item's dependability or feasibility. Therefore, uncertainty regarding the actual level of environmental stress should be considered random. Stress and strength are both treated as random variables in the stress-strength model. Based on the simple stress-strength model, X represents the stress the operating environment places on the unit, and Y represents the unit's strength. The strength of a unit is greater than the stress, so it can perform the intended function. A unit's reliability can be defined as the probability that it will be able to withstand a specified level of stress. In reliability engineering and survival analysis, this model has been found to be of increasing use.

In the development of SS reliability models with single components that follow well-known lifetime distributions, several well-developed estimation techniques have appeared in literature. The EGD is a novel model, a mixture of exponential and gamma distributions. The SS reliability for independent stress and strength random variables that follow the EGD distribution under type-II censoring is discussed in detail in a single-component SS model. The MLE of SS reliability is obtained, and simulations have been done extensively. A comparison is made between the EGD model and the Lindley model based on real-life datasets. A good fit was found for the EGD model, and it can be used when analyzing SS reliability.

For high-reliability products or materials, a prolonged testing period is usually required. The use of ALTs can expedite the testing process. In ALT testing, products are subjected to harsher conditions than they would be under normal use conditions, which reduces their life expectancy. For a variety of reasons, including operational failures, device malfunctions, expense, and time constraints, ALTs may contain censored data.

A simple step stress life testing model with type-II Gumbel lifetime distribution is introduced. A flexible failure rate-based SSALT model is considered based on type-II censoring in this model. Under the concept of a failure rate-based model, point estimates of parameters are described by using the maximum likelihood method. The interval estimation is also derived.

All the research outputs would have given new insights to the reliability and survival analysis researchers.

7.2 Future Directions

Some of the research works that can be addressed in future is given below:

- New distributions using PGDUS transformation can be introduced based on other existing distributions as baseline distributions like inverse Kumaraswamy, inverse Weibull, KM transformation distributions etc.
- Stress-strength reliability estimation of the proposed PGDUS transformed distributions can be addressed.
- Bivariate and multivariate extensions to the proposed BS distribution can be explored in detail.
- Regression models and diagnostics can be developed based on the ν-Birnbaum-Saunders distribution in both uncensored and censored data.
- Bayesian approach to the stress-strength model given can be studied.
- The SSALT models can be designed that utilize type-II Gumbel distributions under censoring schemes such as type-I censoring, hybrid censoring, progressive censoring, etc., using different models such as cumulative exposure model, proportional hazard model, Khamis-Higgins model, etc.
- Inference for the SSALT model in the presence of competing risk model can be introduced.