

# Contents

<b>Declaration</b>	i
<b>Certificate</b>	iii
<b>Certificate</b>	v
<b>Abstract</b>	vii
<b>Acknowledgements</b>	xiii
<b>List of Symbols and Abbreviations</b>	xv
<b>List of Figures</b>	xxv
<b>List of Tables</b>	xxx
<b>1 Introduction</b>	1
1.1 Fluid	1
1.2 Types of fluid flow	1
1.3 Viscosity	3
1.4 Newtonian fluids	4
1.5 Non-Newtonian fluids	4
1.6 Carreau fluid model	5
1.7 Reiner Rivlin fluid	5
1.8 Heat transfer	5
1.9 Mass transfer	7
1.10 Chemical reaction	7
1.11 Magnetohydrodynamics (MHD)	7
1.12 Hall current	9
1.13 Soret effect (Thermophoresis)	9
1.14 Joule heating effect	9

1.15	Porous media . . . . .	10
1.16	Boussinesq approximation . . . . .	11
1.17	Basic equations describing the fluid flow . . . . .	12
1.18	Dimensional analysis & Non dimensional parameters . . . . .	13
1.19	Nanofluid . . . . .	17
1.20	Solution methodology . . . . .	20
1.21	Statistical analysis . . . . .	22
1.22	Response surface methodology (RSM) . . . . .	23
1.23	Sensitivity analysis . . . . .	23
1.24	Literature review . . . . .	23
1.25	Objectives . . . . .	39
<b>2</b>	<b>Statistical analysis of MHD convective ferro-nanofluid flow through an inclined channel with Hall current</b>	<b>41</b>
2.1	Introduction . . . . .	41
2.2	Mathematical formulation . . . . .	41
2.3	Numerical solution . . . . .	45
2.4	Results and discussion . . . . .	47
2.5	Conclusions . . . . .	65
2.6	Appendix . . . . .	66
<b>3</b>	<b>Statistical analysis on MHD convective Carreau nanofluid flow due to bilateral non linear stretching sheet with zero mass flux condition</b>	<b>71</b>
3.1	Introduction . . . . .	71
3.2	Mathematical formulation . . . . .	71
3.3	Numerical solution . . . . .	74
3.4	Result and discussion . . . . .	74
3.5	Statistical Analysis . . . . .	83
3.6	Conclusions . . . . .	88
<b>4</b>	<b>Effects of multi-slip and distinct heat source on MHD Carreau nanofluid flow past an elongating cylinder using statistical method</b>	<b>89</b>
4.1	Introduction . . . . .	89
4.2	Mathematical formulation . . . . .	90

4.3	Numerical solution . . . . .	92
4.4	Result and discussion . . . . .	93
4.5	Statistical Analysis . . . . .	106
4.6	Conclusions . . . . .	109
5	<b>Nanoparticle aggregation kinematics on the quadratic convective MHD flow of nanomaterial past an inclined flat plate with sensitivity analysis</b>	111
5.1	Introduction . . . . .	111
5.2	Mathematical Formulation . . . . .	112
5.3	Numerical solution . . . . .	116
5.4	Results and discussion . . . . .	118
5.5	Response Surface Methodology (RSM) . . . . .	123
5.6	Sensitivity analysis . . . . .	128
5.7	Conclusions . . . . .	131
6	<b>MHD Darcy-Forchheimer hybrid nanoliquid flow over an elongated permeable sheet in a porous medium with hydrodynamic slip constraint</b>	133
6.1	Introduction . . . . .	133
6.2	Mathematical formulation . . . . .	134
6.3	Numerical solution . . . . .	138
6.4	Results and discussion . . . . .	139
6.5	Special Cases . . . . .	148
6.6	Regression Analysis . . . . .	152
6.7	Conclusions . . . . .	156
7	<b>Reiner-Rivlin nanoliquid flow past a spinning disk with Joule heating and non-uniform heat source using Bulirsch-Stoer algorithm</b>	159
7.1	Introduction . . . . .	159
7.2	Mathematical formulation . . . . .	160
7.3	Numerical solution . . . . .	164
7.4	Results and Discussion . . . . .	165
7.5	Response Surface Methodology (RSM) . . . . .	174

7.6 Conclusions . . . . .	183
<b>8 Significance of nanoparticle shape effect on MHD convective alumina-water nanofluid flow over a rotating rigid disk</b>	185
8.1 Introduction . . . . .	185
8.2 Mathematical formulation . . . . .	185
8.3 Numerical solution . . . . .	189
8.4 Results and Discussion . . . . .	194
8.5 Conclusions . . . . .	207
<b>9 Conclusive remarks and future scope</b>	209
Publications in journals and presentations	215
Bibliography	217
References . . . . .	217

# List of Figures

1.1	Laminar and turbulent flows	2
1.2	Flow chart depicting bvp5c routine	21
2.1	Physical configuration of the problem	43
2.2	Variations in Velocity $l'$ with $\phi$	53
2.3	Variations in temperature $\theta'$ with $\phi$	54
2.4	Variations in concentration $C'$ with $\phi$	54
2.5	Variations in velocity $l'$ with $H$	55
2.6	Variations in velocity $l'$ with $M$	55
2.7	Variations in concentration $C'$ with $K_r > 0$	56
2.8	Variations in concentration $C'$ with $(K_r < 0)$	56
2.9	Variations in velocity $l'$ with $S$	57
2.10	Variations in temperature $\theta'$ with $S$	57
2.11	Variations in concentration $C'$ with $S$	58
2.12	Variations in velocity $l'$ with $\lambda$	58
2.13	Variations in temperature $\theta'$ with $\lambda$	59
2.14	Variations in concentration $C'$ with $S_0$	59
2.15	Variations in velocity $l'$ with $\alpha$	60
2.16	Variations in velocity $l'$ with $K_1$	60
2.17	Actual and estimated values of $Cf$	64
2.18	Actual and estimated values of $Nu$	64
2.19	Actual and estimated values of $Sh$	65
3.1	Geometry of the problem	72

3.2 Variation of $f'(\zeta)$ for various values of $\beta^*$	77
3.3 Variation of $g'(\zeta)$ for various values of $\beta^*$	77
3.4 Variation of $f'(\zeta)$ for various values of $We$	78
3.5 Variation of $g'(\zeta)$ for various values of $We$	78
3.6 Variation of $f'(\zeta)$ for various values of $M$	79
3.7 Variation of $g'(\zeta)$ for various values of $M$	79
3.8 Variation of $g'(\zeta)$ for various values of $\delta$	80
3.9 Variation of $\theta(\zeta)$ for various values of $Bi$	80
3.10 Variation of $\theta(\zeta)$ for various values of $S$	81
3.11 Variation of $\theta(\zeta)$ for various values of $Nt$	81
3.12 Variation of $\phi(\zeta)$ for various values of $Nb$	82
3.13 Variation of $\phi(\zeta)$ for various values of $Nt$	82
3.14 Actual and estimated values $Re_x^{-\frac{1}{2}} Nu$ of when $n = 0.7$	87
3.15 Actual and estimated values $Re_x^{-\frac{1}{2}} Nu$ of when $n = 1.7$	88
4.1 Figurative representation	90
4.2 $f'(\zeta)$ for differing $b_1$ values	95
4.3 $\theta(\zeta)$ for differing $\gamma$ values	96
4.4 $f'(\zeta)$ for differing $M$ values	96
4.5 $f'(\zeta)$ for differing $We$ values	97
4.6 $\theta(\zeta)$ for differing $We$ values	97
4.7 $\theta(\zeta)$ for differing $\kappa$ values	98
4.8 $\theta(\zeta)$ for differing $Nb$ values	98
4.9 $\theta(\zeta)$ for differing $Nt$ values	99
4.10 $\phi(\zeta)$ for differing $Nb$ values	99
4.11 $\phi(\zeta)$ for differing $Nt$ values	100
4.12 $\theta(\zeta)$ for differing $Q_T$ values	100
4.13 $\theta(\zeta)$ for differing $Q_E$ values	101
4.14 Parallel effect of $Nb$ and $Nt$ on $Nu(Re_x)^{-\frac{1}{2}}$ when $n = 0.5$	102
4.15 Parallel effect of $Nb$ and $Nt$ on $Nu(Re_x)^{-\frac{1}{2}}$ when $n = 1.5$	102
4.16 Parallel effect of $Q_T$ and $Q_E$ on $Nu(Re_x)^{-\frac{1}{2}}$ when $n = 0.5$	103
4.17 Parallel effect of $Q_T$ and $Q_E$ on $Nu(Re_x)^{-\frac{1}{2}}$ when $n = 1.5$	103
4.18 Parallel effect of $\gamma$ and $\kappa$ on $Nu(Re_x)^{-\frac{1}{2}}$ when $n = 0.5$	104
4.19 Parallel effect of $\gamma$ and $\kappa$ on $Nu(Re_x)^{-\frac{1}{2}}$ when $n = 1.5$	104

---

4.20	Estimated versus actual $Nu(Re_x)^{-\frac{1}{2}}$ when $n = 0.5$	108
4.21	Estimated versus actual $Nu(Re_x)^{-\frac{1}{2}}$ when $n = 1.5$	108
5.1	Physical configuration.	112
5.2	Variation of $S'(\zeta)$ for distinct values of $\phi$ .	119
5.3	Variation of $\theta(\zeta)$ for distinct values of $\phi$ .	120
5.4	Variation of $S'(\zeta)$ for distinct values of $M$ .	120
5.5	Variation of $\theta(\zeta)$ for distinct values of $M$ .	121
5.6	Variation of $S'(\zeta)$ for distinct values of $\alpha_1$ .	121
5.7	Variation of $\theta(\zeta)$ for distinct values of $\alpha_1$ .	122
5.8	Variation of $\theta(\zeta)$ for distinct values of $Q_E$ .	122
5.9	Variation of $\theta(\zeta)$ for distinct values of $R$ .	123
5.10	Surface plot of $Cf_X$ for variation of $\alpha_1$ and $\phi$ .	124
5.11	Surface plot of $Cf_X$ for variation of $\alpha$ and $M$	124
5.12	Surface plot of $Nu_X$ for variation of $\alpha$ and $M$ .	125
5.13	Surface plot of $Nu_X$ for variation of $\alpha_1$ and $\phi$ .	125
5.14	Residual plots for $Nu_X$ .	127
5.15	Contour plots and Response surface plots of $Nu_X$ for all combinations of $\alpha$ , $Q_E$ and $R$ .	129
5.15	Contour plots and Response surface plots of $Nu_X$ for all combinations of $\alpha$ , $Q_E$ and $R$ .	130
5.16	Bar charts depicting the sensitivity of $Nu_X$ with $X_2 = -1, X_2 = 0$ , and $X_2 = 1$	131
6.1	Physical configuration.	135
6.2	Change in $f'(\zeta)$ for differing $\phi_{Cu}$ values.	141
6.3	Change in $f'(\zeta)$ for differing $M$ values.	142
6.4	Change in $f'(\zeta)$ for differing $Fr$ values.	142
6.5	Change in $f'(\zeta)$ for differing $K_1$ values.	143
6.6	Change in $f'(\zeta)$ for differing $b_1$ values.	143
6.7	Change in $g'(\zeta)$ for differing $M$ values.	144
6.8	Change in $g'(\zeta)$ for differing $K_1$ values.	144
6.9	Change in $g'(\zeta)$ for differing $\delta$ values.	145
6.10	Change in $\theta(\zeta)$ for differing $\phi_{Cu}$ values.	145

6.11 Change in $\theta(\zeta)$ for differing $M$ values. . . . .	146
6.12 Change in $\theta(\zeta)$ for differing $Bi$ values. . . . .	146
6.13 Change in $\phi(\zeta)$ for differing $M$ values. . . . .	147
6.14 Change in $\phi(\zeta)$ for differing $K_1$ values. . . . .	147
6.15 Change in $Nu_x(Re_x)^{-\frac{1}{2}}$ for differing $Nt$ and $Bi$ with $\lambda = -0.1$ . . .	150
6.16 Change in $Nu_x(Re_x)^{-\frac{1}{2}}$ for differing $Nt$ and $Bi$ with $\lambda = 0.1$ . . .	150
6.17 Change in $Nu_x(Re_x)^{-\frac{1}{2}}$ for differing $\phi_{Cu}$ and $M$ with $\lambda = -0.1$ . .	151
6.18 Change in $Nu_x(Re_x)^{-\frac{1}{2}}$ for differing $\phi_{Cu}$ and $M$ with $\lambda = 0.1$ . .	151
6.19 Change in $f'(\zeta)$ of the two-dimensional flow for differing $b_1$ values. .	152
6.20 Change in $f'(\zeta)$ of the axisymmetric flow for differing $b_1$ values. . .	153
6.21 Actual $Re_x^{\frac{1}{2}}Cf_x$ versus Estimated $Re_x^{\frac{1}{2}}Cf_x$ with $\lambda = -0.1$ . . . . .	154
6.22 Actual $Re_x^{\frac{1}{2}}Cf_x$ versus Estimated $Re_x^{\frac{1}{2}}Cf_x$ with $\lambda = 0.1$ . . . . .	155
6.23 Actual $Re_y^{\frac{1}{2}}Cf_y$ versus Estimated $Re_y^{\frac{1}{2}}Cf_y$ with $\lambda = -0.1$ . . . . .	155
6.24 Actual $Re_y^{\frac{1}{2}}Cf_y$ versus Estimated $Re_y^{\frac{1}{2}}Cf_y$ with $\lambda = 0.1$ . . . . .	156
7.1 Physical configuration . . . . .	160
7.2 Response in $F'(\zeta)$ for distinct $M$ . . . . .	166
7.3 Response in $G(\zeta)$ for distinct $M$ . . . . .	167
7.4 Response in $\theta(\zeta)$ for distinct $M$ . . . . .	167
7.5 Response in $\phi(\zeta)$ for distinct $M$ . . . . .	168
7.6 Response in $F'(\zeta)$ for distinct $K$ . . . . .	168
7.7 Response in $G(\zeta)$ for distinct $K$ . . . . .	169
7.8 Response in $\theta(\zeta)$ for distinct $K$ . . . . .	169
7.9 Response in $\phi(\zeta)$ for distinct $K$ . . . . .	170
7.10 Response in $\theta(\zeta)$ for distinct $Ec$ . . . . .	171
7.11 Response in $\phi(\zeta)$ for distinct $Sc$ . . . . .	171
7.12 Response in $\theta(\zeta)$ for distinct $A$ & $B$ . . . . .	172
7.13 Response in $\theta(\zeta)$ for distinct $Nt$ . . . . .	172
7.14 Response in $\phi(\zeta)$ for distinct $Nt$ . . . . .	173
7.15 Response in $\theta(\zeta)$ for distinct $\gamma$ . . . . .	173
7.16 Surface plot of $C_f Re^{\frac{1}{2}}$ for variation of $K$ and $M$ . . . . .	174
7.17 Surface plot of $NuRe^{-\frac{1}{2}}$ for variation of $A$ and $B$ . . . . .	175
7.18 Surface plot of $NuRe^{-\frac{1}{2}}$ for variation of $Sc$ and $Ec$ . . . . .	175
7.19 Surface plot of $NuRe^{-\frac{1}{2}}$ for variation of $Nb$ and $Nt$ . . . . .	176

7.20	Residual plots for $Nu_r$ . . . . .	179
7.21	Contour and response surface plot of $Nu_r$ for combinations of $\gamma, K$ .	180
7.22	Contour and response surface plot of $Nu_r$ for combinations of $\gamma, M$ .	180
7.23	Contour and response surface plot of $Nu_r$ for combinations of $M, K$ .	181
7.24	Bar charts depicting the sensitivity of $Nu_r$ . . . . .	182
8.1	Physical configuration . . . . .	186
8.2	Flowchart depicting the numerical scheme. . . . .	191
8.3	Impact of $\phi_{Al_2O_3}$ on $F(\zeta)$ . . . . .	195
8.4	Impact of $\phi_{Al_2O_3}$ on $G(\zeta)$ . . . . .	196
8.5	Impact of $\phi_{Al_2O_3}$ on $\theta(\zeta)$ . . . . .	196
8.6	Impact of $M$ on $F(\zeta)$ . . . . .	197
8.7	Impact of $M$ on $G(\zeta)$ . . . . .	197
8.8	Impact of $M$ on $\theta(\zeta)$ . . . . .	198
8.9	Impact of $M$ on $\phi(\zeta)$ . . . . .	198
8.10	Impact of $b_1$ on $F(\zeta)$ . . . . .	199
8.11	Impact of $b_1$ on $G(\zeta)$ . . . . .	199
8.12	Impact of $b_1$ on $\theta(\zeta)$ . . . . .	200
8.13	Impact of $b_1$ on $\phi(\zeta)$ . . . . .	200
8.14	Impact of $\gamma$ on $\theta(\zeta)$ . . . . .	201
8.15	Impact of $\gamma$ on $\phi(\zeta)$ . . . . .	202
8.16	Impact of $Nt$ on $\theta(\zeta)$ . . . . .	202
8.17	Impact of $Nt$ on $\phi(\zeta)$ . . . . .	203
8.18	Impact of $Nb$ on $\phi(\zeta)$ . . . . .	203
8.19	Impact of $Sc$ on $\phi(\zeta)$ . . . . .	204
8.20	Impact of $\tilde{n}_s$ on $F(\zeta)$ . . . . .	205
8.21	Impact of $\tilde{n}_s$ on $G(\zeta)$ . . . . .	205
8.22	Impact of $\tilde{n}_s$ on $\theta(\zeta)$ . . . . .	206
8.23	Impact of $\tilde{n}_s$ on $\phi(\zeta)$ . . . . .	206



# List of Tables

1.1	Porosity and permeability of some common materials Based on data by (Bejan & Lage, 1991) and (Nield, Bejan, et al., 2006) . . . . .	10
2.1	Thermo physical properties of base fluid and nanoparticles at $25^0$ . .	44
2.2	Amplitude value of the coefficient, $\frac{\varepsilon}{2}e^{it}$ , in the expansion of $(\frac{\partial\theta'}{\partial z})_{z=0}$ with $\lambda = 0, R = 0, S = 0, \phi = 0$ .	48
2.3	Comparison of $(\tau_{0r})$ and $(\beta_0)$ of the steady flow in the absence of $\Omega^*$ , with $G_m = 0, R = 0, K_r = 0, K_1 \rightarrow \infty, S_0 = 0, S = 0, \phi = 0, \alpha = \pi/2$ . . . . .	48
2.4	The Skin friction of $Fe_3O_4$ -water nanofluid when $t = \frac{\pi}{4}, Pr = 6.07, K_r = 1.5, K_1 = 0.7, M = 3.5, S_0 = 0.5, \omega = 20, Sc = 0.22, \alpha = \frac{\pi}{4}, Re = 10$ . . . . .	50
2.5	The Nusselt number of $Fe_3O_4$ -water nanofluid when $t = \frac{\pi}{4}, Pr = 6.07, G_r = 5, G_m = 5, K_r = 1.5, K_1 = 0.7, M = 3.5, H = 1.5, S_0 = 0.5, \omega = 20, Sc = 0.22, \alpha = \frac{\pi}{4}, Re = 10$ .	51
2.6	The Sherwood number of $Fe_3O_4$ -water nanofluid when $Pr = 6.07, G_r = 5, G_m = 5, K_1 = 0.7, M = 3.5, H = 1.5, S = 2, Sc = 0.22, \alpha = \frac{\pi}{4}$ . . . . .	52
2.7	Correlation coefficient ( $r_c$ ), Probable error (PE) and $  \frac{r_c}{PE}  $ values of $Cf$ with respect to the parameters $\phi, H, G_m, S, R, G_r$ and $\lambda$ . . . . .	53
2.8	Correlation coefficient ( $r_c$ ), Probable error (PE) and $  \frac{r_c}{PE}  $ values of $Nu$ with respect to the parameters $\phi, S, R$ and $\lambda$ .	61
2.9	Correlation coefficient ( $r_c$ ), Probable error (PE) and $  \frac{r_c}{PE}  $ values of $Sh$ with respect to the parameters $\phi, R, S_0, \lambda, \omega, t$ and $K_r$ .	61

2.10 Linear regression model for $Cf$ , with Number of observations :35, Error degrees of freedom:27, Root mean square error :0.00461, $\mathcal{R}$ -squared:0.989, Adjusted $\mathcal{R}$ -Squared 0.986, F-statistic vs. constant model:332, p-value =1.7e-24 . . . . .	62
2.11 Linear regression model for $Nu$ with number of observations :20, Error degrees of freedom:15, mean square error :0.0713, $\mathcal{R}$ -squared: 0.995, Adjusted $\mathcal{R}$ - Squared 0.994, F-statistic vs. constant model:773, p-value =3.59e-17 . . . . .	62
2.12 Linear regression model for $Sh$ , Number of observations :35, Error degrees of freedom:27, Root mean square error :0.00753, $\mathcal{R}$ -squared:0.998, Adjusted $\mathcal{R}$ -Squared 0.998, F-statistic vs. constant model:2.31e+03, p-value =8.37e-36 . . . . .	63
 3.1 Skin friction coefficients, local Nusselt number and local Sherwood number are defined as given below: . . . . .	75
3.2 Comparison of $f''(0)$ and $g''(0)$ when $\beta^* = 1$ and $M = 0$ . . . . .	76
3.3 Variation in local Nusselt number at $\zeta = 0$ when $m = 1$ , $Nt = 1$ , $S = 0.2$ , $M = 0.5$ , $We = 3$ , $\delta = 0.8$ , $\beta^* = 0.1$ , $Nb = 0.5$ and $Bi = 0.4$ . . . . .	84
3.4 Variation in Skin Friction Coefficients at $\zeta = 0$ when $m = 1$ , $Nt = 1$ , $S = 0.2$ , $M = 0.5$ , $We = 3$ , $\delta = 0.8$ , $\beta^* = 0.1$ , $Nb = 0.5$ and $Bi = 0.4$ . . . . .	85
3.5 Variation in Skin Friction Coefficients at $\zeta = 0$ when $m = 3$ , $Nt = 1$ , $S = 0.2$ , $M = 0.5$ , $We = 3$ , $\delta = 0.8$ , $\beta^* = 0.1$ , $Nb = 0.5$ and $Bi = 0.4$ . . . . .	86
3.6 Correlation Coefficient ( $r_c$ ), Probable Error ( $PE$ ) and $  \frac{r_c}{PE}  $ of reduced Nusselt number at $\zeta = 0$ when $Bi = 0.4$ and $Nb = 0.5$ . . . . .	86
 4.1 Comparison of $Nu(Re_x)^{-\frac{1}{2}}$ with $We = \kappa = M = Q_T = Q_E = Nt = b_1 = \gamma = b_3 = 0$ and $Nb \rightarrow 0$ . . . . .	92
4.2 Comparison of $-\frac{1}{2}Cf(Re_x)^{\frac{1}{2}}$ with $M = Q_T = Q_E = Nt = b_1 = \gamma = b_3 = 0$ and $Nb \rightarrow 0$ . . . . .	94

4.3 Variation in $- \frac{1}{2} C_f (Re_x)^{\frac{1}{2}}$ {at $\zeta = 0$ } when $\kappa = 0.2$ , $We = 2$ , $M = 1$ , $m = 0.5$ , $Q_T = 0.02$ , $Q_E = 0.04$ , $Nt = 0.1$ , $Nb = 0.4$ , $b_1 = 0.2$ , $\gamma = 0.2$ and $b_3 = 0.2$ . . . . .	105
4.4 Variation in $Sh(Re_x)^{-\frac{1}{2}}$ {at $\zeta = 0$ } when $\kappa = 0.2$ , $We = 2$ , $M = 1$ , $m = 0.5$ , $Q_T = 0.02$ , $Nt = 0.1$ , $Q_E = 0.04$ , $Nb = 0.4$ , $b_i = 0.2$ , $\gamma = 0.2$ and $b_3 = 0.2$ . . . . .	106
4.5 Probable error ( $PE$ ) and correlation coefficient ( $r_c$ ) of $Nu(Re_x)^{-\frac{1}{2}}$ . . . . .	107
 5.1 Thermophysical properties of the nanofluid with aggregation (Mackolil & Mahanthesh, 2021b) . . . . .	114
5.2 Thermo physical properties of base fluid and nanoparticles at 300K (Mackolil & Mahanthesh, 2021b) . . . . .	115
5.3 Comparison of $-\theta'(0)$ values and when $\alpha_1 = \alpha = M = Q_E = R = 0$ and $\phi = 0$ with the results of (Kuznetsov & Nield, 2010) and (Mahanthesh & Mackolil, 2021) . . . . .	117
5.4 Comparison of $S''(0)$ when $\alpha_1 = 0.1, m = 0.5, M = 0.1, Q_E = 0.2, R = 1$ and $\phi = 1\%$ . . . . .	117
5.5 The effective levels of parameters . . . . .	126
5.6 ANOVA table . . . . .	126
5.7 The sensitivity values of the response $Nu_X$ when $X_1 = 0$ . . . . .	130
 6.1 The effective thermophysical properties of the hybrid nanoliquid (with alumina as nanoparticle 1 and copper as nanoparticle 2) . . . . .	137
6.2 Thermophysical properties of water, alumina, and copper (Aziz, Jamshed, Aziz, Bahaidarah, & Ur Rehman, 2021), (Aaiza, Khan, & Shafie, 2015), (Hussanan, Salleh, Khan, & Shafie, 2017) . . . . .	138
6.3 Resemblance of $-Re_x^{\frac{1}{2}} C_f_x$ and $-Re_y^{\frac{1}{2}} C_f_y$ for differing values of $K_1$ , $Fr$ , and $\delta$ when $M = Nt = \lambda = \phi_{Al_2O_3} = \phi_{Cu} = b_1 = Bi = 0$ , $Pr = Sc = 1$ , and $Nb \rightarrow 0$ . . . . .	139
6.4 Resemblance of $Re_x^{-\frac{1}{2}} Nu_x$ for differing values of $K_1$ , $Fr$ , and $\delta$ when $M = \lambda = \phi_{Al_2O_3} = \phi_{Cu} = b_1 = 0$ , $Pr = Sc = 1$ , $Bi = 0.3$ , $Nt = 0.2$ , and $Nb = 0.5$ . . . . .	139
6.5 Comparison on $Re_x^{\frac{1}{2}} C_f_x$ for differing values of $Fr$ , $M$ , $K_1$ , $\phi_{Cu}$ , and $b_1$ when $Nt = 0.5$ , $Nb = 0.5$ , $\delta = 0.1$ , $Bi = 0.5$ , $\phi_{Al_2O_3} = 0.02$ . . . . .	148

6.6 Comparison on $Re_y^{\frac{1}{2}}$ $Cf_y$ for differing values of $M$ , $K_1$ , $\phi_{Cu}$ , $b_1$ and $\delta$ when $Nt = 0.5$ , $Nb = 0.5$ , $Fr = 0.5$ , $Bi = 0.5$ , $\phi_{Al_2O_3} = 0.02$ . . . . .	149
7.1 Comparison of $F''(0)$ , $G'(0)$ , $F(\infty)$ , and $-\theta'(0)$ with $K = 0$ , $M = 0$ , $Ec = 0$ , $A = 0$ , $B = 0$ , $Nt = 0$ , $Sc = 0$ , $Nb \rightarrow 0$ , $Pr_f = 0.72^a$ , and $Pr_f = 6^b$ . . . . .	165
7.2 The effectual levels of parameters. . . . .	177
7.3 Experimental (numerical) design. . . . .	177
7.4 ANOVA table . . . . .	178
7.5 The sensitivity values of the response $Nu_r$ when $X_1 = 0$ . . . . .	183
8.1 Effective nanofluid constants see (Timofeeva, Routbort, & Singh, 2009), (Mustafa, Khan, Hayat, & Alsaedi, 2018), (Brinkman, 1952). . .	188
8.2 Thermophysical Properties of water and $Al_2O_3$ (see (Wakif & Sehaqui, 2022)). . . . .	189
8.3 Nanoparticle shape properties of $Al_2O_3$ (see (Timofeeva et al., 2009)).	190
8.4 Validation for regular fluids ( $Pr = 6.2$ , $\phi_{Al_2O_3} = 0$ , $M = 0$ , $b_1 = 0$ , $\gamma = 0$ , $\zeta_\infty = 30$ , $\tilde{N} = 100$ ) . . . . .	193
8.5 Validation for nanofluids ( $Pr = 7$ , $A_1 = A_2 = A_3 = A_4 = A_5 = 1$ , $Sc = 5$ , $Nb = 0.5$ , $Nt = 0.5$ , $b_1 = 0$ , $\gamma = 0$ , $\zeta_\infty = 15$ , $\tilde{N} = 100$ ) . . .	193
8.6 Numerical estimation of $C_{fr}$ for various shapes of nanoparticles ( $M = 0.2$ , $Sc = 5$ , $Nb = 0.2$ , $Nt = 0.1$ , $b_1 = 0.25$ , $\gamma = 0.25$ , $\zeta_\infty = 30$ , $\tilde{N} = 100$ ) . . . . .	194
8.7 Numerical estimation of $Nu_r$ for various shapes of nanoparticles ( $M = 0.2$ , $Sc = 5$ , $Nb = 0.2$ , $Nt = 0.1$ , $b_1 = 0.25$ , $\gamma = 0.25$ , $\zeta_\infty = 30$ , $\tilde{N} = 100$ ). . . . .	195