CHAPTER 1

General Introduction

Fluid- the material which exhibits continuous deformation under the action of shearing forces, frequently classified as liquids or gases. The substances having intermolecular forces, thus possess definite volume are liquids, while fluid with extremely small intermolecular forces are gases. Fluid mechanics is the stream of science that deals with the behavior of stationary and moving fluids. The analysis of fluid properties at rest is the fluid statics, whereas the study of fluid in motion, in absence of pressure force, is the fluid kinematics. And with considering pressure force, the stream of science that describes the characterization of fluid in motion is fluid dynamics.

The analysis of moving fluid is necessary as it is existing in all nature activities and our daily life. The notion of, a continuous medium developed by Aristotle, and the physical law of buoyancy initiated by Archimedes was the first step in the study of fluid flow. Sir Isaac Newton's study of fluid statistics and fluid dynamics paved the way for the analysis of fluid flow through mathematical and quantitative physics. Later Daniel Bernoulli elucidated the significance of fluid pressure and based on his notion, Euler developed elementary equations of hydrodynamics for ideal fluid.

Fluid dynamics provides a wide range of applications in transportation, industry, manufacturing, aeronautical engineering, bio-medical field, etc. It also contributes in measure the evolution of planets, weather patterns, ocean tides, plate tectonics, and also blood circulation. Most of the natural motions obey conservation laws, which are treated as fundamental principles of fluid dynamics. The mathematical analysis of the fluid dynamic system is significant in engineering.

Computational fluid dynamics is a principal branch of fluid dynamics, where the numerical and analytical interpretation of fluid flow is considered. Mathematical software Matlab, Mathematica etc. are used to obtain more accurate solutions to fluid flow problems, and the accuracy of such software and numerical methods authenticates by comparing the values with previous studies. Computational fluid dynamics is important in research and engineering problems, such as aerospace and aerodynamics analysis, environment engineering, biological engineering, design and analysis of heat exchangers, and engines, and so on.

Hence the purpose of this thesis is to analyze some problems in magnetohydrodynamic nanofluid flow in channels and past plates with the impacts of radiation, viscous dissipation, heat source or sink, chemical reaction, and so on.

1.1 SOME BASIC PROPERTIES

1.1.1 Density

The density is the mass per unit volume. The density of a fluid ρ at a point P is defined as

$$\rho = \lim_{\delta V \to 0} \delta m / \delta V \tag{1.1.1}$$

Where δV is the volume element around P, and δm is the mass of the fluid within δV.

1.1.2 Pressure

Pressure of a fluid is the fraction of normal force to the area. When a fluid is contained inside a surface, it exerts a force at each point of the surface. The pressure of a fluid p at a point P is defined as

$$p = \lim_{\delta A \to 0} \delta F / \delta A \tag{1.1.2}$$

δA is the elementary area around P and δF is the normal force due to the fluid on δA.

1.1.3 Specific heat

The amount of heat needed to raise the temperature of one gram of a material by one Celcius degree. If Q is the heat energy, m-mas, and δT-the temperature difference, then specific heat capacity is

$$C = \frac{Q}{(m\delta T)} \tag{1.1.3}$$

Specific heat at constant pressure is denoted by C_p

1.1.4 Viscosity

The viscosity of a fluid is a tool for the resistance of the fluid to flow. It is also known as internal friction. Simply it is the ratio of shear stress (the tangential force per unit area when fluid is moving) to shear rate (velocity gradient).

$$\mu = \frac{shear\ stress}{velocity\ gradient} \tag{1.1.4}$$

The effect of viscosity on the motion of a fluid is determined by kinematic viscosity

$$\vartheta = \frac{\mu}{\rho} \tag{1.1.5}$$

1.2 CLASSIFICATION OF FLUIDS

1.2.1 Ideal and Real fluids

The non-viscous fluids are called ideal fluids. When a fluid is moving shear stress is zero in an ideal fluid, hence there is no resistance in the motion of the fluid. Generally, ideal fluids are not existing in nature. The viscous fluids are called real fluids. Nonzero shear stress exists always in real fluids when fluid is in motion, hence some resistance occurs in a fluid motion.

1.2.2 Compressible and incompressible fluid

The ability to change in volume of a mass of fluid is known as compressibility. Gas, vapor, steam, etc. are regarded as compressible fluids, as they have no definite volume. On the other hand, liquids are considered to possess a definite volume, hence they are incompressible fluids.

1.2.3 Newtonian and non-Newtonian fluid

Fluid flow is in such a manner that the various layers of the fluid slide uniformly over one another. Sir Isac Newton has proved that the tangential stress acting on one layer of the fluid by the other is directly proportional to the relative velocity and inversely proportional to the distance between them. Ifu and $u + du$ are the velocities of two adjacent layers of fluid separated by a distance dy, by Newton's law of viscosity, the tangential stress τ is

$$\tau = \mu \frac{du}{dy} \tag{1.2.1}$$

Where μ is the coefficient of viscosity. The fluids which obey this relation are referred to as Newtonian fluids. Water, vegetable oils, milk, air, and all gases are Newtonian fluids in usual circumstances.

The fluids which do not obey Newton's law of viscosity are non-Newtonian fluids. They show a non-linear correlation between shear stress and shear rate. Cake batter, polymer solutions, blood, sand in water, paste, and silicone oil are some examples of non-Newtonian fluids.

1.3 TYPES OF FLOW

1.3.1 Laminar and Turbulent flow

In fluid flow, if the individual streamlines run parallelly in a well-ordered manner, it is laminar flow. Blood flow through capillaries and oil flow through a thin tube are laminar flow. In turbulent flow, the streamlines are intertwined with each other and run improperly. Most of the flows seen in nature such as lava flow, atmosphere, and ocean current, are turbulent flows.

1.3.2 Steady and unsteady flow

The flow with time-independent fluid properties is known as steady flow. That is

$$\frac{\partial P}{\partial t} = 0, \tag{1.3.1}$$

where $P(x, y, z)$ is any property such as density, velocity, temperature, and so on. The flow-through pipes, pumps, nozzles, etc. are examples of steady flow. The flow in which the fluid properties depend on time is the unsteady flow.

1.4 HEAT TRANSFER

Heat transfer is the physical process that concerns the swapping of thermal energy between material bodies due to temperature differences. By the second law of thermodynamics, heat transfer occurs from a higher temperature region to a lower temperature region, until two regions attain the same temperature. The heat may be transferred in three modes.

1.4.1 Conduction

Heat conduction is a process in which heat transfer takes place from particles to particles by their collision. These particles do not necessarily have displacement from their position, but they vibrate against each other. These vibrating molecules interact with the neighboring molecules and transfer heat energy.

Thermal conductivity

It correlates the heat flow to the temperature gradient. If T and $T + dT$ are the temperatures of two adjacent layers of the fluid separated by a distance dy, by Fourier's law of heat conduction, the quantity of heat q transferred through a unit area in unit time is

$$q = -k\frac{dT}{dy} \tag{1.4.1}$$

k- the coefficient of thermal conductivity

The effect of conductivity on the temperature field is determined by thermal diffusivity,

$$\alpha = \frac{k}{\rho C_p} \tag{1.4.2}$$

1.4.2 Convection

Convection is a heat transfer phenomenon by the movement of the fluid between temperature differed areas of the fluid. Convection problems are classified as:

Free and Forced convection

Free convection is a mechanism, in which the fluid motion is generated only by gravity effect on heated fluids of variable density, not by any external source, also termed natural convection. The lighter (less dense) components rise, while heavier (denser) components will fall leading to a bulk fluid movement that creates natural convection.

If the motion is due to an external force (such as a fan, pump), it is forced convection. Based on forced convection, devices are developed for heat transfer in all types of heat exchangers. The convection in which both free and forced convection mechanism occurs is called mixed convection.

Bioconvection

Bioconvection occurs due to the upward-directed swimming of motile microorganisms, that are a little denser than water. The upper surface of the fluid becomes dense due to the upward movement of microorganisms, thus a hydromagnetic instability arises under some conditions that create bioconvective flow.

1.4.3 Radiation

The heat energy transfer from an object, having a higher temperature, in the form of electromagnetic waves is thermal radiation. For example, radiation from hot gas in outer space, infrared radiation emitted by an electric heater, etc. Radiation does not require a material medium for energy transfer. Thermal radiation is formed when the heat energy, by the movement of charges in material transformed into electromagnetic waves. According to Stefan- Boltzman total energy radiated per second per unit area is directly proportional to the fourth power of the absolute temperature of the surface.

$$E_t = \sigma_t^* T^4 \tag{1.4.3}$$

where T is the absolute temperature and σ_t^* is the Stefan- Boltzman constant.

1.5 MASS TRANSFER

Mass transfer is the process of movement of the mass because of the concentration difference of species in a mixture. The molecular motions generate mass transfer to reduce the concentration difference at distinct points within the mixture of two or more components. Mass transfer is mainly classified as mass diffusion and convective mass transfer.

1.5.1 Mass Diffusion

Mass diffusion is a mass transfer process in a medium at rest, in which the driving force is the concentration difference between neighboring regions in the medium.

1.5.2 Convective Mass Transfer

If the mass transfer occurs between a boundary surface and a moving fluid or between two relative moving fluids, it is known as a convective mass transfer.

Chemical Reaction

A chemical reaction is a process in which the transmission of one or more substances or reactants to one or more different substances takes place. The transmitter substances are either chemical elements or compounds. Encouraging or defeating convection has a significant role in heat transfer problems. Sustaining a non-uniform temperature gradient is an effective mechanism to delay or advance the beginning of convection. Chemical reactors have a vital role to establish a non-uniform temperature gradient. Adding a chemical reactant promotes the transformation of substances into another. If the density of the reactant is different from the density of the product formed, an isothermal reaction can cause free convection.

1.6 MAGNETOHYDRODYNAMICS

Magnetohydrodynamics (MHD); also called magneto-fluid dynamics or hydromagnetics, is the physical and mathematical framework that deals with the dynamics of electrically conducting fluids such as saltwater, ionized gases or plasmas (solar atmosphere), electrolytes, and liquid metals (molten iron, mercury, gallium) under the influence of a magnetic field.

The important observations from the historical evolution of magnetohydrodynamics are: Euler (1701-1780) generalized Newton's second law of dynamics to a continuous medium with no internal shear stress and Navier (1785-1836) modified Euler's equation corresponding to uniform viscous fluid. Stokes (1819-1903) presented the concept of internal shear stress, its mathematical frame led to the well-known Navier-Stokes equation. On the other hand in electromagnetism, Volta (1745-1827) innovated the Voltaic pile or battery. Ampere (1775-1836) found a connection between magnetism and electricity. Later Ohm (1789-1854) established the electrical conduction law and Faraday (1791-1867) discovered that the difference in magnetic flux across a loop generates variation in electric potential. Then Maxwell (1831-189) developed four equations by extending Faraday's work.

Gauss Law :

$$divD = \rho_e \tag{1.6.1}$$

Ampere's Law :

$$curlH = J \tag{1.6.2}$$

Faraday's law :

$$curlE = -\frac{\partial B}{\partial t} \tag{1.6.3}$$

Solenoidal property :

$$divB = 0 \tag{1.6.4}$$

Generalized Ohm's Law :

$$J = \sigma\left(E + v \;\times B\right) \tag{1.6.5}$$

where $B = \mu H \ and \ D = \varepsilon_d E$

Here v denotes the velocity vector, H is the magnetic field,B is the magnetic induction, μ is the magnetic permeability, E is the electric field, J is current density, ρ_e is free electric charge density, D is the dielectric field, ε_d is the dielectric constant and σ is the electrical conductivity of the fluid.

Hartmann in 1937 analyzed the steady magnetic flux on moving liquid metal and realized thin boundary layers. Later in 1942, Alfven explained the dynamics of electrically conducting liquids indicated as magnetohydrodynamics or MHD. Hence the exploration of MHD was raised by Hennes Alfven, and Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism are together used to describe magnetohydrodynamics.

The basic theory beyond MHD is that, in a moving conductive fluid, subject to an external magnetic field, an electric current and an induced magnetic flux formed. The interaction of the original magnetic field and induced magnetic flux creates body force. More precisely, consider the flow of electrically conducting fluid with velocity v across the magnetic field of magnetic flux B. Thus a current density

$$J = \sigma\left(v \;\times B\right) \tag{1.6.6}$$

induced in this field and the interaction of this current with existing magnetic flux B produces electromagnetic body force,

$$F = J \times B \tag{1.6.7}$$

known as Lorentz force, where σ is the electrical conductivity.

The area that describes the dynamics of electrically conducting fluids in the presence of electric and magnetic fields is termed electromagnetohydrodynamics (EMHD). By generalized Ohm's law, the induced current density due to electric field E and magnetic field B is

$$J = \sigma \left(E + v \ \times B \right) \tag{1.6.8}$$

1.7 POROUS MEDIUM

The material consisting of a solid matrix with frequently distributed pores (void spaces) in it is a porous medium or a porous material. The void spaces allow the flow of fluid through these materials. The distribution of pores is irregular in a natural porous medium. How the voids are placed in the medium, their interrelation, shape, and location distinguish the porous medium. The porous medium is characterized by permeability and porosity.

1.7.1 Porosity

Porosity is the measure of pores in a medium. It is the ratio of the volume of voids to the total volume. That is porosity ϵ of a porous medium is

$$\epsilon = \frac{V_v}{V_T} \tag{1.7.1}$$

where V_v is the volume of fluid occupied in void space and V_T is the total volume of the medium (including the empty space in the medium).

1.7.2 Permeability

Permeability is the measure of the ability of a medium to transmit the fluid through it. Obviously, it is related to the size of void spaces distributed in the medium. The fluid motion in a porous medium is evaluated in terms of volume or average movement of the fluid elements over regions of space.

According to Darcy law, the flow through a porous medium is proportional to the applied pressure gradient.

$$\delta p = -\frac{\mu}{K} u \tag{1.7.2}$$

where u is the Darcy velocity and K is the permeability of the material.

1.8 BASIC EQUATIONS

The fundamental principles of fluid dynamics are mathematically interpreted based on mass, momentum, and energy conservation laws; accordingly, the total mass, linear momentum, and energy of a closed system remain constant. The fluid flow is ruled by continuity, motion, and energy equations which are derived from the above-mentioned conservation laws.

1.8.1 Equation of continuity

The equation of continuity is reached from the law of mass conservation. Considering the fluid of density ρ flow with velocity $\vec{V}$, the continuity equation is given by

$$\frac{\partial \rho}{\partial t} + div\left(\rho \vec{V}\right) = 0 \tag{1.8.1}$$

1.8.2 Navier –Stokes equation

The equation of motion also known as Navier-Stokes' equation or momentum equation obtained from the law of momentum conservation. Consider the fluid of density ρ flow with velocity $\vec{V}$, let $\vec{F}$ be the body force per unit mass, p is the pressure and μ is the coefficient of viscosity, the Navier-Stokes' equation is given by

$$\rho\left(\frac{\partial \vec{V}}{\partial t} + \left(\vec{V}.\nabla\right)\vec{V}\right) = \rho\vec{F} - \nabla p + \mu\nabla^2\vec{V} \tag{1.8.2}$$

1.8.3 Equation of energy

The energy equation is obtained from the law of energy conservation. Consider the fluid with density ρ, temperature T, then the equation of energy is given by,

$$\rho C_p\left(\frac{\partial T}{\partial t} + \left(\vec{V}.\nabla\right)T\right) = k\frac{\partial}{\partial x_i}\left(\frac{\partial T}{\partial x_i}\right) \tag{1.8.3}$$

Where k-coefficient of thermal conductivity, C_p- specific heat

1.9 BOUNDARY LAYER

The elementary equations of hydrodynamics for inviscid fluid, developed by Euler were insufficient to explain flow separation, the pressure loss in a channel flow, and the drag force that occurs on a system moving in a fluid. Later Navier and Stokes modified Euler's equation for viscous fluids, which are known as Navier-Stokes equations, but it was difficult to make these equations analytically solved form. The boundary layer theory proposed by Ludwig Prandtl overcame the inability of Navier-Stokes equations and gives a physically strong explanation of the significance of viscosity in the evaluation of flow separation and frictional drag.

A thin layer neighboring the bounding surface, where velocity, temperature, and concentration gradients normal to the surface are important within this layer and negligible above this layer, is the boundary layer of a fluid flow. The significant classifications of boundary layers are:

1.9.1 Velocity Boundary Layer

Consider the fluid flow over a flat plate. The fluid particles have zero velocity when it attaches to the surface. These particles resist the movement of particles in the neighboring layer and the same process continues for the adjacent layers. This sequel becomes slight at a distance $y = \delta$ from the surface. This detain of fluid movement is due to the shear stress τ acting in planes that are collateral to the fluid velocity. As y values from the surface increase, the $x-$ component of velocity u enhances and approaches the free stream velocity u_∞.

Here δ is the value of y such that $u = 0.99\ u_\infty$ and is called boundary layer thickness. The velocity boundary layer profile mentions that u differs with y through the boundary layer. The velocity gradient and shear stresses are greater within this

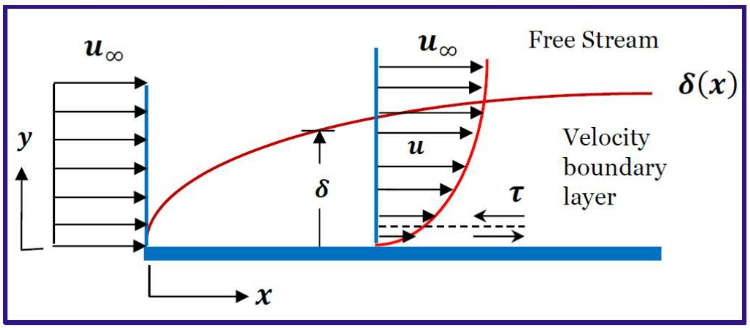

Figure 1.1: *Velocity boundary layer development on a flat plate*

layer and negligible outside this layer. As reaching away from the leading edge, the influence of viscosity creeps into the free stream and the boundary layer raises.

1.9.2 Thermal Boundary Layer

The thermal boundary layer develops due to the temperature differences in the fluid-free stream and surface. Consider the fluid flow past an isothermal flat surface with temperature T_s. Let T_∞ be the uniform ambient temperature. The fluid particles lying close to the surface receive the temperature from the surface and interchange the thermal energy with the fluid particles in the adjacent layers and generate temperature gradients in the fluid. The region in which these temperature gradients exist is the thermal boundary layer. The thermal boundary layer thickness δ_t is the value of y such that $\frac{(T_s - T)}{(T_s - T_\infty)} = 0.99$. As reaching away from the leading edge, the influence of heat transfer creeps into the free stream and the thermal boundary layer grows.

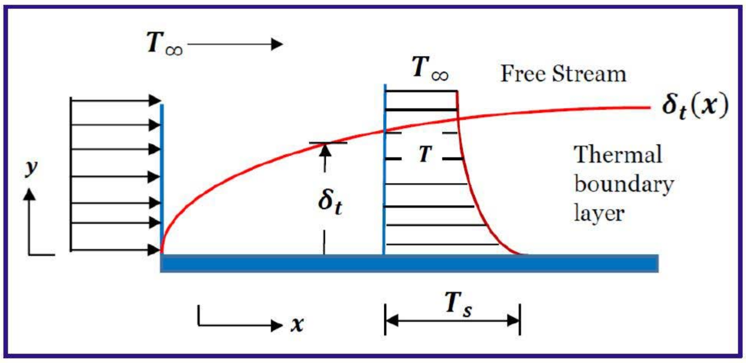

Figure 1.2: *Thermal boundary layer development on a flat plate*

1.9.3 Concentration Boundary Layer

The concentration boundary layer defines convection mass transfer. Consider the flow of fluid, which is the mixture of chemical species A and B, over a flat plate. Let $C_{(A,S)}$ and $C_{(A,\infty)}$ denote the concentration of species A at the surface and in the fluid-free stream respectively. If $C_{(A,S)}$ is differ from $C_{(A,\infty)}$, a concentration boundary layer generates. The region in which these concentration gradients exist is the concentration boundary layer. And the concentration boundary layer thickness δ_c is the value of y such that $\frac{(C_{(A,S)} - C_A)}{(C_{(A,S)} - C_{(A,\infty)})} = 0.99$.

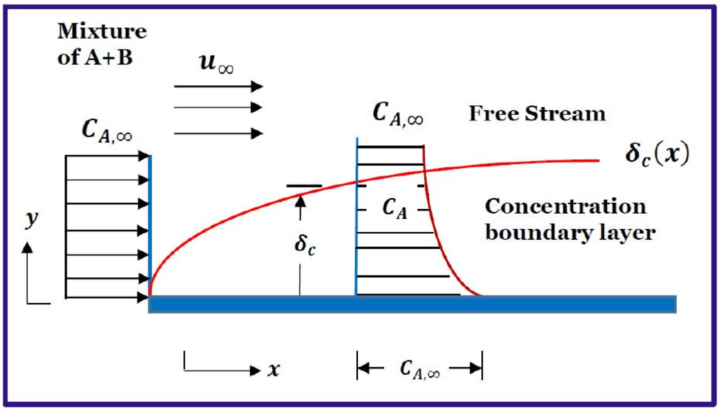

Figure 1.3: *Concentration boundary layer development on a flat plate*

1.10 PHYSICAL QUANTITIES

Physical quantities are used to evaluate friction and heat and mass transfer behavior between the boundary and the fluid. They are:

1.10.1 Drag Coefficient (Cf)

The drag coefficient is the proportion of skin shear stress at the wall to the dynamic pressure of a free stream. It measures the frictional force at the boundary between fluids and walls.

$$Cf = \frac{\tau_w}{\rho U^2} \tag{1.10.1}$$

where τ_w is the wall shear stress.

1.10.2 Nusselt Number (Nu)

It gives an idea about the movement of heat from a solid surface to a fluid. Nusselt number describes the correlation between thermal energy and thermal conductivity by the fluid movement and is given by,

$$Nu = \frac{q_w x}{k\left(T_w - T_\infty\right)} \tag{1.10.2}$$

where q_w-surface heat flux,
k-thermal conductivity,
x- the distance from the leading edge.

1.10.3 **Sherwood Number** (Sh)

It conveys the correlation between mass energy and mass diffusivity.

$$Sh = \frac{q_m x}{D_B \left(C_w - C_\infty\right)} \tag{1.10.3}$$

where q_m- Mass flux,
D_B-Mass diffusion coefficient

1.10.4 **Microorganism Density Number** (Nn)

It is the fraction of convective microbial transfer to the microorganism diffusion coefficient.

$$Nn = \frac{q_n x}{D_m \left(N_w - N_\infty\right)} \tag{1.10.4}$$

where q_n-Motile microorganism flux,
D_m-Microorganism diffusion coefficient

1.11 NON-DIMENSIONAL QUANTITIES

A research work attains its value when it is put into related implementations, and a successful solicitation is obtained after several experiments. Using geometrically similar small models instead of original full-scale objects reduces the cost of experiments. The dynamical similarity is significant here. Two fluid flows are dynamically similar if the values of the dimensionless parameters in both flows remain the same. The methods for obtaining the dimensionless parameters in a flow problem are inspectional analysis and dimensional analysis.

In the inspectional analysis method, the non-dimensional parameters are obtained by reducing the governing equations of the flow to a dimensionless form. It is possible, only if we have an entire set of equations that describe the flow.

An alternative method, with which the non-dimensional parameters may be formed from the physical quantities occurring in a flow problem is known as dimensional analysis. They are derived based on the dimensions in which each of the quantities involved in a phenomenon is expressed, and hence, must not depend on the units chosen for the calculations. The advantage of dimensional analysis is that the equations are not necessary and one is rewarded according to insight and cleverness.

The important non-dimensional parameters are:

1.11.1 **Prandtl number** (Pr)

The proportion of momentum diffusivity to the thermal diffusivity of the fluid is referred to Prandtl number.

$$Pr = \frac{\mu C_p}{k} = \frac{\vartheta}{\alpha} \tag{1.11.1}$$

It measures the significance of thermal conduction and viscosity of the fluid. Higher values of Pr quantify the greater viscosity and lower thermal conductivity of the fluid. Ethylene glycol is a fluid with higher Pr.

1.11.2 **Schmidt number** (Sc)

It is the ratio of kinematic viscosity (ϑ) to the mass diffusivity D_B. That is

$$S_c = \frac{\vartheta}{D_B} \tag{1.11.2}$$

Sc correlates the relative hydromagnetic and mass transfer boundary layer thickness. And greater Sc values constitute lower chemical molecular diffusivity.

1.11.3 **Eckert number** (Ec)

It is significant to regulate the correlation between kinetic energy and boundary layer enthalpy difference in the fluid system. Also, it narrates the heat dissipation. It is defined as

$$Ec = \frac{U^2}{C_p\left(T_w - T_\infty\right)} \tag{1.11.3}$$

1.11.4 **Reynolds number** (Re)

It is the most significant non-dimensional parameter in fluid dynamics which is supportive to identify a laminar or turbulent flow. A Higher Reynolds number specifies the domination of turbulent flow over the laminar flow. Reynolds number is defined as

$$Re = \frac{Ineretia\ force}{Viscous\ force} = \frac{UL}{\vartheta} \tag{1.11.4}$$

1.11.5 **Grashof number** (Gr)

It measures the correlative significance of buoyancy force to the viscous force and specifies free convection. Grashof number is defined by,

$$Gr = \frac{gL^3\left(T_w - T_\infty\right)}{\vartheta^2 T_\infty} \tag{1.11.5}$$

1.11.6 **Modified Grashof number** (Gm)

Similar to Gr, it occurs in the free convection problem, when the mass transfer is considered. And is defined as

$$Gm = \frac{gL^3\left(C_w - C_\infty\right)}{\vartheta^2 C_\infty} \tag{1.11.6}$$

1.11.7 **Magnetic field parameter**(H)

It represents the correlation between electromagnetic force and viscous force. It is defined as

$$H = \sqrt{\frac{\sigma B_0^2 L^2}{\mu}} \tag{1.11.7}$$

Hartmann introduced this term, which is also known as the Hartmann number. The Lorentz force is generated at higher H values.

1.11.8 **Lewis number** (Le)

The Lewis number is defined as the ratio of thermal diffusivity and mass (molecular) diffusivity and it plays an important role in the simultaneous effect of heat and mass transfer processes. The Lewis Number is also the ratio of the Schmidt Number and Prandtl Number and it may also be expressed as

$$Le = \frac{\alpha}{D_B} = \frac{\vartheta/D_B}{\vartheta/\alpha} = \frac{Sc}{Pr} \tag{1.11.8}$$

1.12 NANOFLUIDS

The colloidal suspension of ultrafine particles (1-100 nm) with a base fluid is termed nanofluid. The nanoparticles used in nanofluids are typically made of metals (like Ag, Ti, Cu, Fe, etc), oxides (like Fe_3O_4, Al_2O_3, TiO_2, etc), carbides (like CaC_2, SiC, etc), or carbon nanotubes, while the base fluids include water, oil, and ethylene glycol.

Nanoparticles possess distinct physical properties based on shape, size, dispersion state, crystallinity, and surface properties, and hence possess diverse applications in various fields. For example, potassium, silicon, iron, silver, zinc oxide, etc. are the nanoparticles employed in agricultural fields. Also, zinc oxide, silver, titanium oxide, gold, etc. are employed in cosmetics.

However, the study of nanofluids overcomes the limited thermal conductivity of regular fluids, hence the researchers focus their attention on hybrid nanofluids, the modern class of nanofluids. The colloidal suspension of two or more nanomaterials with a base fluid is termed as hybrid nanofluid. The collaborative effect of nanoparticles can improve the thermal properties of the fluid. Water-based $Al_2O_3 - Fe_3O_4, CNT - Fe_3O_4, TiO_2 - Ag$ hybrid nanofluids analyzed in this thesis.

1.12.1 Thermal properties of nanofluid

Thermophysical characteristic is an important factor to be considered in choosing nanofluids for heat transfer implementations. The effective thermophysical properties of the nanofluids are mixed properties of the base fluid and solid particles. Nanofluids are not a simple mixture of nanoparticles to the base fluid. Its thermophysical properties depend on, the methods used to prepare nanofluids and under what conditions they are prepared. Tiwari and Das developed a simpler nanofluid model considering the effective thermophysical properties of a nanofluid.

Effective Dynamic Viscosity:

$$\frac{\mu_{nf}}{\mu_f} = \frac{1}{(1-\phi)^{2.5}} \tag{1.12.1}$$

Effective Density:

$$\frac{\rho_{nf}}{\rho_f} = (1-\phi) + \phi\left(\frac{\rho_s}{\rho_f}\right) \tag{1.12.2}$$

Effective Electrical Conductivity:

$$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3\left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f} + 2\right) - \left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi} \tag{1.12.3}$$

Effective Coefficient Of Thermal Expansion:

$$\frac{\beta_{nf}}{\beta_f} = (1-\phi) + \phi\left(\frac{\beta_s}{\beta_f}\right) \tag{1.12.4}$$

Effective Specific Heat:

$$\frac{(\rho C_p)_{nf}}{(\rho C_p)_f} = (1-\phi) + \phi\left(\frac{(\rho C_p)_s}{(\rho C_p)_f}\right) \tag{1.12.5}$$

Effective Thermal Conductivity:

$$\frac{K_{nf}}{K_f} = \frac{K_s + 2K_f - 2\phi\,(K_f - K_s)}{K_s + 2K_f + 2\phi\,(K_f - K_s)} \tag{1.12.6}$$

If P is a property, P_f, P_s, and P_{nf} correspond to, the property of the base fluid, solid particles, and nanofluid.

$$\phi = \frac{V_{np}}{V_f} \tag{1.12.7}$$

is the volume fraction of nanoparticles, where V_{np} and V_f denote the nanoparticle volume and base fluid volume respectively.

1.13 METHODOLOGY

The mathematical tools utilized to find the numerical solutions for the non-linear partial differential equations representing the governing flow are:

- Perturbation Technique
- Matlab bvp4c and bvp5c solver of the finite difference scheme

1.13.1 Perturbation Technique

The non-steady boundary layer problems essentially entail a steady flow, but with a minute non-steady disturbance. As this disturbance (perturbation) is supposed to be small compared with the steady flow, for the steady perturbation, the equations can be split into non-linear equations.

The necessary suggestion is that the formulation has a small parameter ε, in either the governing equation or in the boundary conditions. In a flow having a high Reyolds number $\varepsilon = \frac{1}{Re}$, in a creeping flow $\varepsilon = Re$, and in the flow around an airfoil ε is the ratio of thickness to chord length. In the perturbation technique, the solutions are assumed to be a series of ε, in which the higher order terms act as a perturbation on the lower order terms.

$$f\,(x,y,t) = \; f_0\,(x,y) + \sum_{i=1}^{\infty} \varepsilon^i f_i\,(x,y,t) \tag{1.13.1}$$

When this assumed solution is substituted in the governing flow problem, and the resulting terms are ordered by the powers of ε, a set of differential equations are obtained. All the sets of differential equations except zeroth order will be linear and can be solved.

1.13.2 bvp4c and bvp5c

bvp4c and bvp5c are computing programs advanced by MATLAB software to solve boundary value problems. It can be used to solve a large class of two-point boundary value problems of first-order ordinary differential equations, which is in general form:

$$y'\left(x\right) = f\left(x, y\left(x\right), p\right) \tag{1.13.2}$$

subject to the boundary conditions,

$$g(y(a), y(b), p) = 0 \tag{1.13.3}$$

where f is a continuous function in y, p is a vector of unknown parameters, and $a \leq x \leq b$. bvp4c and bvp5c are executed in the same way, but with different error tolerances. These codes have only three arguments: a function odes for evaluating ordinary differential equations, a function bcs for evaluating the residual in the boundary conditions, and a structure solinit that delivers a guess on a mesh and the solution on this mesh.

As the boundary conditions are coded $g(y(a), y(b)) = 0$, instantly $y(a)$ is approximated as ya, and $y(b)$ is approximated as yb by the function bcs, which assesses and gives back the residual $g(ya, yb)$. When solving a boundary value problem, a hypothesis of solution should be given to identify and compute the solution. The bvp4c or bvp5c solver assesses the hypothesis as a structure generated by the auxiliary function bvpint. The initial assertion of the function bvpint is a hypothesis for a mesh that discloses the nature of the solution. The next assertion is a hypothesis for the solution on the defined mesh, in which the solution contains a pair of elements $y(x)$ and $y'\left(x\right)$. The hypothesis is given in two ways, the hypothesis for $y(x)$ has an accurate shape and meets the boundary conditions or considers all the approximated solution elements to be constants and provides the vector of these constants immediately. The latter is the convenient way and often works.

The base stone of the bvp4c or bvp5c is the notion of residual, which influence the error control and mesh selection. The fundamental method to find a solution is centered on polynomial collocation with four Lobatto points for bvp4c and five Lobatto points in the case of bvp5c. The amount to be approximated and managed is residual for bvp4c and residual and error for bvp5c.

1.14 STATISTICAL ANALYSIS

The further scrutinization of numerical results obtained by the above-mentioned methods utilizing statistical techniques has a significant role in the research world due to its efficiency in producing accurate quantitative results. The statistical techniques measure the interconnection of the parameters which cannot be determined using the conventional numerical procedure. Some important statistical techniques used in this thesis are:

1.14.1 Correlation

Correlation explains the dimension and direction of interconnection with two or more variables. The correlation coefficient delineates the nature of the relationship between an independent and a dependent variable, ranging between -1 and 1. The sign of the correlation coefficient represents the nature of dependency of the independent variable on the dependent variable. If the independent variable isn't related to the dependent variable by any means then the correlation coefficient returns the value 0. Let $x_1, x_2, \ldots\ldots, x_n$ are independent variables and $y_1, y_2, \ldots\ldots, y_n$ are corresponding dependent variables, then the correlation coefficient is

$$r = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (y_i - \bar{y})^2}} \tag{1.14.1}$$

where $\bar{x}$and $\bar{y}$ are means of $x_1, x_2, \ldots\ldots, x_n$ and $y_1, y_2, \ldots\ldots, y_n$ respectively.

1.14.2 Probable error

Probable error (PE) is a strong statistical measure to check the reliability of the calculated coefficient. It is measured as

$$PE = \left(\frac{1 - r^2}{\sqrt{n}}\right) 0.6745 \tag{1.14.2}$$

where n denotes the number of observations. The correlation is said to be significant if $\left|\frac{r}{PE}\right| > 6$.

1.14.3 Regression analysis

Regression analysis helps to recognize the parameters, which are influencing a quantity of interest. It determines which parameters are most effective, which parameters can be neglected, and also how these variables are affected altogether. In regression analysis, a model is developed that is expressed by a function and determines a connection between a dependent variable and an independent variable is the regression model. Out of different regression models, the linear model is considered to be the best model, in which linear predictor functions are used to model the relationships.

1.14.4 Response Surface Methodology (RSM)

RSM is a statistical technique based on an experimental design that measures the mutual impact of effectual parameters (independent variables) on the response variable (dependent variable). Experimental trials are costly and time-consuming whereas RSM reduces the number of trials and optimizes the response variable. RSM estimates a model for the quantity of interest using the Central Composite Design (CCD) experimental data. RSM using CCD is a suitable sequential experimentation method which is incorporated in the present study with 20 $(2^j+2j+o, j = 3, 0 = 6)$ runs, here o denotes the number of faces and j denotes the number of factors. The relation involves 2^3 factorial 6 center and 6 axial points. A full quadratic 3-level factorial design is adopted in the study as follows

$$\begin{aligned} \mathit{Response}\ \ (Y) = L_1X_1 + L_2X_2 + L_3X_3 + L_4X_1X_2 + L_5X_2X_3 + L_6X_1X_3 \\ + L_7X_1^2 + L_8X_2^2 + L_9X_3^2 + L_{10}. \quad (1.14.3) \end{aligned}$$

Here $L_i (i = 1, 2, ...10)$ denote the regression coefficients. The RSM technique becomes advantageous in finding the levels of the parameters that optimize the response. ANOVA table evaluates the quality of the fitted quadratic model in a reliable way. The significance of linear, quadratic, and interactive regression terms are computed statistically using F and p values. The regression terms in the fitted model are significant for high value of F and if $p < 0.05$.

1.14.5 Sensitivity analysis

Sensitivity analysis estimates the range and nature of dependency indicated by the parameters on the targeted variable. In other words, sensitivity analysis accounts for the variation induced by the augmenting parameter on the remaining influential parameter. The sign of sensitivity indicates the nature of the relationship between the targeted variable and the effectual parameters, and the magnitude of sensitivity indicates the intensity of the effect on the quantity of interest.

1.15 OBJECTIVES

The main objective of this research work are:

- To characterize the flow properties of nanofluids in various situations.
- To study the impact of some external effects on flow properties and physical quantities of MHD nanofluid flow.
- To Ensure the reliability of the theoretical investigations regarding the physical quantities of choice with statistical techniques.
- To explore the applications of nanofluids in bio-medical and industrial field.

1.16 REVIEW OF RELEVANT LITERATURE

1.16.1 NANOFLUID

The dynamic with regular heat transfer fluids faces a lack of thermal conductivity, which is a major drawback in engineering applications. In industries such as manufacturing, microelectronics, transport, and defense, the cooling of machinery and equipment is a major issue. The conventional thermal transport fluids such as water, oil, etc. are not sufficient to meet these cooling needs, due to their low ability to conduct heat. Even though many mechanisms have been suggested to enhance the heat transfer ability of regular fluids, the technique of adding ultrafine solid particles in a conventional fluid, introduced by Choi and Eastman is a good solution to the existing trouble. Nanofluids have a significant role to upgrade and stabilize thermophysical properties such as thermal diffusivity, thermal conductivity, convective heat transfer coefficients, and viscosity of the fluid which boosted its acceptance. Gradually the researchers explored the influence of nanofluids in renewable energy (solar thermoelectric devices, solar collector, biomass, and geothermal), biomedical engineering (chromatography, neuro electronic links, Vivo therapy, photodynamic therapy, and drug delivery), commercial production (plastics, ceramics, metals, paints), and food processing.

Nanofluids have been a hot topic for a while now. Using nanofluids instead of the normal base fluids has yielded many awarding results. It is seen that nanofluids have better convective heat transfer capabilities relevant in industrial, engineering and medical fields. Food processing, nuclear power plants, heating or cooling, nanomagnet switches, separate oil and gas in the reservoir, solar collectors, and electronic devices are some of the industrial and engineering applications. Blood diagnostics, antibacterial, photodynamic imaging and therapy, drug loading and delivery, biosensors and cancer therapy are some significant implementations of nanofluid in biotechnology.

(Choi & Eastman, 1995) first introduced an innovative class of fluids, the nanofluid, developed by suspending metallic nanoparticles to conventional fluids. (Eastman, 1999) studied the thermal properties of nanofluids.(Lee, Choi, Li, , & Eastman, 1999) generated oxide nanofluids and investigated their thermal characteristics. They noticed that nanofluids with a minute amount of nanoparticles possess comparatively larger thermal conductivity. (Xuan & Li, 2000) initiated the theoretical evaluation of the thermal properties of nanofluid. They also scrutinized the convective heat transfer and properties of nanofluid flow in a tube (Xuan & Li, 2003). (Gosselin & da Silva, 2004) highlights the significance of, optimizing the thermal properties of the free and forced convective nanofluid flow. (Akbarinia, 2008) explored the drag factor and heat transfer rate due to nanofluid flow along a pipe. (Hamad & Bashir, 2009) introduced the boundary layer flow of a non-Newtonian fluid past a stretching surface. (Abu-Nada & Eiyad, 2010) reviewed the thermal conductivity and viscosity effects of nanofluid flow under natural convection.

Numerous scientists focus their attention on nanofluids which revolutionized the field of fluid dynamics and the studies are still going on. (Kulkarni & Das, 2012) analyzed properties of nanofluids theoretically and experimentally. (Murshed & De Castro, 2014) explain the methodology for developing nanofluids, their characteristics, and principles to employ nanofluids in important areas. (Kiyani, Hayat, Ahmad, Waqas, & Alsaedi, 2021) semi-analytically investigated the bidirectional Williamson nanofluid flow in porous space and observed that space and temperature-dependent heat sources have a positive effect on temperature. (Kumar, Bhattacharyya, Seth, & Chamkha, 2021) studied magnetite water-based nanofluid flow over a rotating disk in the presence of an external magnetic field and Arrhenius activation energy and noted that the thermophoresis parameter hurts heat transfer rate. (Hazarika, Ahmed, & Chamkha, 2021) used the fourth-order RK-shooting technique to numerically investigate the MHD flow of a chemically reacting water-based nanofluid over a permeable stretching sheet involving chemical reaction, thermophoresis, heat source, and viscous dissipation. (Seth, Bhattacharyya, Kumar, & Chamkha, 2018) examined the unsteady hydromagnetic boundary layer flow of a thermally radiating nanofluid considering Navier's velocity slip and external magnetic field past a non-linear stretching sheet.

The thermophysical properties of a nanofluid have a significant role in analyzing nanofluid flow and improving its potential utilities. (Tiwari & Das, 2007) developed a nanofluid model to characterize the behavior of nanofluids by considering the solid volume fraction of nanoparticles. (Roşca & Pop, 2017) used Tiwari Das nanofluid model to study stagnation point flow over a stretching or shrinking surface. (Eid & Nafe, 2022) employed Tiwari and Das nanofluid model to analyze the MHD nanofluid flow through an exponentially stretching surface.

1.16.2 HYBRID NANOFLUID

The latest nanotechnology research works focus on finding practices that help in boosting the efficiency and transfer properties of the considered nanoliquid. This is where hybrid nanoliquid (colloidal suspension of two or more nanomaterials) comes into the scene. The collaborative effect alters the nanoliquid's heat transfer rate proving to be beneficial in many engineering and industrial fields (like solar energy systems, car radiators, nuclear system cooling, micromanufacturing processes, etc.).

(Jana, Salehi-Khojin, & Zhong, 2007) were the first to experimentally analyze hybrid nanofluid in 2007. (Sundar, Singh, & Sousa, 2014) investigated hybrid nanofluid's friction factor and convective heat transfer coefficient. (Suresh, Venkitaraj, Selvakumar, & Chandrasekar, 2011) investigated the thermophysical properties of water-based $Al_2O_3 - Cu$ hybrid composite and observed that viscosity and thermal conductivity of the prepared composite can be enhanced by adding nanoparticles. Heat transfer aspects of regular and hybrid nanofluid flow over a stretching sheet numerically analyzed by (Devi & Devi, 2016). (Waini, Ishak, & Pop, 2019b) characterized hybrid nanofluid flow past a stretching or shrinking sheet. (Manjunatha, Kuttan, Jayanthi, Chamkha, & Gireesha, 2019) explored the impact of magnetic field and variable viscosity in improving the heat transfer ability of hybrid nanofluid flow.

(Junoh, Ali, Arifin, Bachok, & Pop, 2020) numerically explored the consequence of induced magnetic field (IMF) on the heat transfer and hydromagnetic stagnation point flow over a lengthening and shortening sheet and revealed that the heat transfer rate was higher for the hybrid nanoliquid. (Acharya & Mabood, 2021) employed the fourth-order Runge-Kutta method to numerically inspect the hybrid nanoliquid flow over a slippery permeable bended structure. They perceived that the hybrid nanoliquid exhibits lower drag coefficient and higher Nusselt number. (Waini, Ishak, & Pop, 2019a) explored the two-dimensional steady flow of water-based Al_2O_3-Cu hybrid nanoliquid over a nonuniformly shrinking/stretching permeable surface. (Ali, Asjad, & Akgül, 2021) remarked that the water-based hybrid nanoliquid showcased improved nanofluid temperature and velocity when compared with engine oil-based nanofluid.

(Khashi'ie, Arifin, Pop, & Nazar, 2021) examined the influence of gyrotactic microorganisms in hybrid nanofluids and distinguished higher heat and mass transfer rates and motile density in hybrid nanofluid compared with respective regular nanofluids. (Shahsavar, Saghafian, M.R.Salimpour, & Shafii, 2016)studied the thermal conductivity and viscosity of the hybrid nanofluid and pointed out that hybrid nanofluids act as shear thinning fluid. They concluded among nanoparticles, Carbon nanotubes provide excellent thermal, chemical, mechanical, and electrical properties. Owed to the capability of moving through various body tissues and probe into cells $CNTs$ are auspicious nanostructures and used in therapeutic drug delivery system is its most desirable (Masotti & Caporali, 2013)(Bhirde et al., 2009). Because of the immense thermal conductivity of CNT and the strong magnetic medium of Fe_3O_4, their combination assured an amazing future in nanoscience. (Kaiser, Buerki-Thurnherr, & Wick, 2013) noticed that without being affected by all provisions single wall carbon nanotubes can be in contact with organisms and perchance absorbed by them and also observed that the reduced cell adhesion of cells exposed to $SWCNT$, have no major side effects on cell functions. (Raza et al., 2020) analyzed the refinement of heat transfer in fluid flow with different $CNTs$. For cooling purposes in industries combination of CNT and Fe_3O_4 nanoparticles are very useful (Mohebbi, Izadi, Delouei, & Sajjadi, 2019). Compared with $CNT-H_2O$ nanofluid,$CNT - Fe_3O_4$ hybrid nanofluid with H_2O as base fluid possess higher thermal conductivity and more drag coefficient. Moreover, $CNT - Fe_3O_4$ hybrid nanofluid is a promising platform for magnetically engaged anti-cancer drug delivery (Fan, Jiao, Gao, Jin, & Li, 2013). However the instability of nanofluid, a limitation of applications in bio-science overcome by adding microorganisms.

Titanium dioxide (TiO_2) is an effective photocatalyst due to its strong oxidizing power, non-toxicity, and long-term chemical and physical stability. TiO_2 nanofluids provide various applications in energy systems. With the advancement of heat and mass transfer processes, the sector of application of TiO_2 nanofluids expands to the fields of heat pipes, solar collectors, energy storage refrigeration, and other energy applications. The thermal conductivity of the TiO_2 nanofluids can be affected by the ingredients of the base fluids. (Chen, Witharana, Jin, Kim, & Ding, 2009) found that the effective thermal conductivity of TiO_2 nanoparticles with ethylene glycol as base fluids are higher than that with water as a base fluid. (Duangthongsuk & Wongwises, 2009) investigated the heat transfer coefficient and friction factor of water-based TiO_2 nanofluid within a horizontal double-tube countercurrent heat exchanger under turbulent flow conditions. They found that the nanofluid heat transfer coefficient depends on the nanofluid, the nanofluid temperature, and the mass flow rate of the hot water. (Reddy & Chamkha, 2016) made a comparison study of MHD convective flow of TiO_2 -water and Al_2O_3 -water nanofluids. (Acharya, Das, & Kumar Kundu, 2016) studied the effect of variable thickness on the steady two-dimensional boundary layer flows of a TiO_2-water and Ag-water nanofluid past a slendering stretching sheet.

Silver nanoparticles are stable, long-lasting, and subject to controlled release. Silver-doped materials are chemically durable and release silver ions for a long time period. (Berger, Spadaro, Chapin, & Becker, 1976) observed that silver solutions and silver metal act as powerful antimicrobial agents for centuries, owing to a broad spectrum of antibacterial activity as well as low toxicity towards cells. Several studies (Dai & Bruening, 2002), (Yuranova et al., 2006) have been reported to explain the indirect effect of silver on bacteria. Silver nanoparticles perform as an excellent antibacterial coating in the food industry, water disinfection, and other disinfection-related fields. (Zhang et al., 2003) discussed the significance of combining silver and TiO_2. it is observed that 2-4 nano meter-sized silver nanoclusters are strongly anchored to the TiO_2 nanoparticles with high dispersion, and also, suspending silver nanoparticles improves the photocatalytic and bactericidal activities of TiO_2.

1.16.3 MAGNETOHYDRODYNAMICS

Magnetohydrodynamics (MHD), the area that deals with the study of dynamics of electrically conducting fluids under the influence of magnetic field has raised quite an interest over the years due to its versatile number of application in various fields; like in geophysics, engineering, biomedical engineering, magnetic drug targeting and many others. Magnetic drug targeting in cancer therapy, which involves establishing more explicit ways to transport cancer-killing drugs to affected areas without harm to surrounding tissues, is one of the significant implementations of magnetohydrodynamics.

(Williams, 1930) described the electromotive forces developed due to a magnetic field in a moving fluid. (Greenspan & Carrier, 1959) studied the MHD steady flow of a viscous incompressible electrically conducting fluid. (Van Blerkom, 1960) mathematically analyzed the MHD viscous fluid flow over a sphere. (Soundalgekar & Murty, 1980) numerically analyzed the thermal properties of magnetohydrodynamic flow with injection, suction, and pressure gradient. (Tezer-Sezgin & Köksal, 1989) employed the finite element method to explore the velocity and induced magnetic field of MHD steady flow through a rectangular pipe and observed that wall conductivity has an inverse proportion with flux. (Ishak, Nazar, & Pop, 2008) conducted a numerical study on the thermal transfer of magnetohydrodynamic flow due to a stretching cylinder. (Hayat, Ahmed, Sajid, & Asghar, 2007) employed homotopy analysis to study second grade fluid flow through a porous channel in presence of the magnetic field.

MHD is commonly paired with convective flows which can either be natural, forced or mixed. Natural or free convection is the type of flow where the motion is not generated by an external source. Natural Convection has called in a lot of attention from researchers with its applications ranging from engineering to nature. (Jha & Aina, 2016) theoretically explored influence of IMF on free convective flow in a vertical microchannel and reported that augmenting magnetic Prandtl number and Hartmann number causes a decrease in volume flow rate. (Dash & Ojha, 2018) discussed about the MHD viscoelastic fluid flow betwixt two infinite horizontal permeable plates involving sinusoidal pressure gradient and noted a decline in velocity profile on amplifying Hartmann number.

1.16.4 ELECTROMAGNETOHYDRODYNAMICS (EMHD)

The realm that deals with the dynamics of electrically conducting fluids under the sway of the electric and magnetic fields are termed electromagnetohydrodynamics (EMHD). Numerous learners have prospected the significance of EMHD due to its relevancy in geophysics, micropumps, magnetic drug targeting, microscale devices, and biomedical engineering. (Liu, Jian, & Tan, 2018) conducted an entropy generation analysis of EMHD flow in a curved rectangular microchannel. They perceived that the electric field parameter had a positive impact on the entropy generation rate. An investigation on the EMHD behavior of a third-grade fluid modeled using the Darcy-Brinkman-Forchheimer model was conducted by (Zhang, Bhatti, & Michaelides, 2020). (Zainal, Nazar, Naganthran, & Pop, 2021) reviewed the unsteady EMHD flow of water-based Al_2O_3-Cu hybrid nanofluid. They detected an exaggeration in the heat transfer characteristics due to the electric and magnetic parameters. (Shah, Bonyah, Islam, & Gul, 2019) investigated the EMHD rotational flow of kerosene oil-based CNTs nanofluid over a stretching sheet and observed an ascending nature of velocity with electric field parameters. (Daniel, Aziz, Ismail, Bahar, & Salah, 2019) numerically examined the EMHD convective nanofluid flow past a permeable stretching sheet imposed with stratification. They concluded that electric and magnetic field parameters exhibit an opposite flow behavior to velocity and temperature. (Abbas, Hayat, Ayub, Bhatti, & Alsaedi, 2019) analyzed the impact of the electrical and magnetic field in nanofluid flow suspended with gyrotactic microorganisms past a porous Riga plate.

1.16.5 POROUS MEDIUM

A porous plate corresponds with a plate having frequently distributed void spaces in it. They are found to be beneficial in chemical engineering (for filtration and clarification), agricultural engineering, (in the study of underwater resources), and the petroleum industry (to study the movement of natural gas, oil and water). (Muskat, 1937) was among the first to learn about the fluid flow past a porous media mathematically. (Lapwood, 1948) observed the occurence of convective fluid flow through a porous medium. (Fulks, Guenther, & Roetman, 1971) discussed and derived conservation laws for the fluid flow through a porous medium. (Vafai & Thiyagaraja, 1987) studied the heat transfer and flow properties of fluid flow through the regions connecting two distinct porous media, a porous medium and fluid area, and a porous medium and an impermeable medium. (Patil & Kulkarni, 2008) investigated the impact of chemical reaction and heat source on a free convective flow past a porous medium.

(Nayak, Akbar, Tripathi, & Pandey, 2017) numerically analyzed the three dimensional hydromagnetic nanoliquid flow through an exponentially lengthening porous sheet with the aid of the fourth order Runge-Kutta method. They noticed a rise in velocity with augmenting porosity parameter values. (Das, Tarafdar, & Jana, 2018) inspected the impact of the transverse magnetic field, slip condition and Hall current on an unsteady hydromagnetic rotating flow over a periodically accelerated horizontal porous plate. They noted that increasing Hall current has a positive effect on the velocity profile close to the plate and a negative effect on the velocity profile away from the plate. (Biswal, Chakraverty, Ojha, & Hussein, 2021) elucidated the influence of the transverse magnetic effect on the flow of water based fluid with added silver and copper nanoparticles in a semi-porous channel utilizing the least square method.

1.16.6 **BIOCONVECTION**

The upward-directed swimming of microorganisms in a suspension due to its lower density than water forms an unbalanced dense surface layer that initiates the bioconvective flow. The presence of microorganisms enhances the stability of the fluid that plays a significant role in biotechnology, bio-microsystems, and bio-nano coolant systems. (Kuznetsov & Avramenko, 2004) were among the first few to investigate the stability of a suspension containing microorganisms and small particles. (Uddin, Khan, Qureshi, & Anwar Bég, 2017) employed the Runge-Kutta-Fehlberg method to numerically survey the impact of thermal and hydrodynamic slip constraints on the water-based bio-nanomaterial containing microorganisms. They observed that the augmenting bioconvection Peclet number tends to improve the microorganism density number. The consequence of multiple slips on bioconvective flow was studied by (Alshomrani, Ullah, & Baleanu, 2020). (Bhatti, Shahid, Abbas, Alamri, & Ellahi, 2020) studied the role of activation energy on a suspension of nanoparticles and microorganisms in a magnetized fluid with the aid of successive local linearization methods.

(Pal & Mondal, 2018a) elucidated the bioconvective MHD nanofluid flow past an exponentially stretching surface and observed a decline in the motile density due to augmenting values of microorganism concentration difference parameter and bioconvection Peclet number. The relevance of three-dimensional bioconvective flow due to an exponentially stretching has been explored by (Alqarni, Waqas, Alghamdi, & Muhammad, 2022). They noticed that the microbial concentration ascends with microorganism Biot number but descends with Peclet number. (M. Khan, Salahuddin, Malik, Alqarni, & Alqahtani, 2020) and (Shafiq, Sindhu, & Khalique, 2020) numerically analyzed the bioconvective tangent hyperbolic nanofluid flow past an exponentially stretching sheets.

1.16.7 **VISCOUS DISSIPATION**

Viscosity is the resistance experienced by a fluid to a change in shape, or movement of neighboring portions relative to one another. It plays an important role in chemical engineering, polymer industry and lubrication processes. The viscous dissipation effect serves a major role in geophysical flows, industrial applications, and aerodynamic heating. (Lund, Omar, Raza, & Khan, 2021) illustrated the dual solutions for dissipative MHD hybrid nanofluid flow past a stretching/shrinking surface and noticed an escalation in the temperature with augmenting values of the Eckert number. (Azhar, Iqbal, & Maraj, 2019) implemented a fractional approach to investigate the consequence of viscous dissipation on the stagnation-point Jeffery fluid flow. The characteristic of viscous dissipation on micropolar fluid flow over a nonlinear stretched sheet has been elucidated by (Patel & Singh, 2019). It is observed that the Eckert number accelerates fluid flow and heat transfer process. (Hazarika et al., 2021) discussed the impact of viscous dissipation on water-based nanofluids over stretching surfaces in the presence of chemical reaction.

1.16.8 STRATIFICATION

The formation of layers due to the variations in mass, heat, and motile density profiles or the presence of discrete fluids is called stratification. (Yang, Novotny, & Cheng, 1972) studied the natural convection flow from a vertical flat plate, submerged in a temperature stratified environment. The consequence of stratification effects on hydromagnetic mixed convective nanoliquid flow was investigated by (Alsaedi, Khan, Farooq, Gull, & Hayat, 2017) with the aid of the homotopy analysis method (HAM). They observed a drop in the nanoliquid temperature with a hike in the thermal stratification parameter. (Ahmad, Nadeem, Muhammad, & Issakhov, 2020) elucidated the effect of double stratification on carbon nanotube (CNT) nanofluid flow and detected a decrease in the nanofluid temperature and volume fraction concerning thermal and solutal stratification parameters, respectively. (Ramzan, Riasat, Shah, Kumam, & Thounthong, 2020) examined the impact of thermal stratification parameter on unsteady ethylene glycol-based nanoliquid flow.The stratification effects on bioconvective stretched Maxwell nanofluid flow analyzed by (Ijaz Khan, Waqas, Hayat, Imran Khan, & Alsaedi, 2017) and observed a reduction in concentration and temperature profiles by the respective stratification parameters. (Naz, Tariq, & Alsulami, 2020) studied Walter's B nanofluid stratified flow consisting of swimming microorganisms.

1.16.9 THERMAL RADIATION

The phenomenon of thermal radiation has crucial importance in space technology and intense heat processes such as nuclear power plants, gas turbines, gas-cooled nuclear reactors, and space vehicles. (Viskanta & Grosh, 1962) are concerned about the impact of thermal radiation on a boundary layer flow. (Chamkha, 2000) studied the radiative hydromagnetic flow over a permeable surface. (Gireesha, Umeshaiah, Prasannakumara, Shashikumar, & Archana, 2020) executed the behavior of radiation in MHD three-dimensional Jeffrey fluid flow through a stretching sheet and observed an increment in fluid temperature and nanoparticle concentration with the changes in the radiation parameter. (Nasir et al., 2018) implemented the homotopy analysis method (HAM) to address thermal radiation and thermophoresis effects on the three-dimensional rotating nanofluid flow containing single-wall carbon nanotubes (SWCNT). They noticed that the nanofluid temperature is an expanding function of radiation parameter. The significant influence of radiation parameters on heat and mass transfer rate is considered by (Besthapu, Haq, Bandari, & Al-Mdallal, 2019). (Gireesha, Archana, Prasannakumara, Gorla, & Makinde, 2017) and (Ramesh, Prasannakumara, Gireesha, Shehzad, & Abbasi, 2017) analyzed the radiative Casson nanofluid and Maxwell fluid flows due to a stretching surface, respectively. Further, the micropolar fluid flow due to a stretching surface considering radiation effect has been studied by (Anantha Kumar, Sugunamma, & Sandeep, 2019).

1.16.10 **HEAT SOURCE**

Convection induced by internal heat sources has been broadly studied because of its wide range of applications in astrophysics, geophysics, fire and combustion modeling, thermal ignition, miniaturization of electronic components, etc. In such flows, the buoyancy force is incremented due to the heat source resulting in modification of heat/mass transfer characteristics. Heat sources explore the hydro-thermal integrity of the flow. (Noghrehabadi, Saffarian, Pourrajab, & Ghalambaz, 2013) conducted an entropy analysis for nanofluid flow past a stretching sheet in the presence of heat generation/absorption and partial slip. it is analyzed that the increase of the heat generation parameter reduces the entropy generation number in the vicinity of the sheet. (2017) theoretically explored the MHD Oldroyd-B nanofluid flow with the heat source or sink induced by a stretching sheet. (Saba et al., 2018) probed the heat transfer analysis of nanofluid flow over a curved stretching surface under the influence of a heat source and a reduction in the magnitude of the local heat flux rate for increasing values of heat generation parameter is observed. (Sandeep & Sulochana, 2018) proposed a new mathematical model for analyzing the momentum and heat transfer behavior of Maxwell, Jeffrey, and Oldroyd-B nanofluids past a stretching surface in the presence of heat source/sink and observed an enhancement in thermal boundary layer thickness with a heat source or sink parameters. (Kotha, Kolipaula, Venkata Subba Rao, Penki, & Chamkha, 2020) studied the impact of internal heat generation on bioconvective MHD nanofluid flow and detected a reduction in heat transfer rate with the heat generation parameter increments.

1.16.11 FLUID FLOW THROUGH CHANNELS

The flow between two parallel plates is a common topic. (Singh, 1999) theoretically analyzed the viscous fluid flow between two parallel porous plates. (Ganesh & Krishnambal, 2006) considered the steady laminar flow of a viscous fluid between two parallel porous plates. (Sweet, Vajravelu, Van Gorder, & Pop, 2011) explore two-dimensional MHD viscous fluid flow between two moving parallel plates.

With the introduction of nanofluids, the major drawbacks against improving thermal properties by using conventional fluids were overcome. The widespread implementation of nanofluids accelerates all related areas. Many more research works on the nanofluid flow between two parallel plates have been carried out within this small period. (Sheikholeslami & Ganji, 2013) numerically analyzed the thermal properties of nanoliquid flow between parallel plates. The nanoliquid flow in presence of a magnetic field between parallel plates is reviewed in (Sheikholeslami, Hatami, & Domairry, 2015). (Mohyud-Din, Zaidi, Khan, & Ahmed, 2015) studied mass and heat transfer of nanoliquid flow rotating parallel plates. MHD nanoliquid flow in a semi-porous channel was explored by (Sheikholeslami, Hatami, & Ganji, 2013). A lot of studies have been done on hybrid nanoliquid flow between parallel plates, in a very short period. (Ramesh, Madhukesh, Prasannakumara, & Roopa, 2022) elucidated heat transfer aspects of hybrid nanofluid flow with heat source/sink and chemical reaction. The proficiency of hybrid nanoliquid flow between two parallel plates is examined by (Khashi'ie, Waini, Arifin, & Pop, 2021).

Many researchers have analyzed the hydromagnetic flow between two vertical porous plates due to free convection and varying geometrical shapes. However, only a countable number of works involving MHD-free convective flow between two vertical porous plates moving in different directions have been published. Previous studies were based on conventional (base) fluids. (Singh & Mathew, 2009b) studied three-dimensional MHD fluctuating free convective flow between two vertical porous plates moving in opposite directions. (V. Gupta, Jain, & Jha, 2016) theoretically analyzed the convective effects and heat transfer aspects of MHD flow of viscous fluids, between two vertical plates moving in opposite directions, where the channel was partially filled with a porous medium. An ascending temperature distribution and a descending main flow velocity are observed with the enhancement of the magnetic field parameter. (U. Gupta, Jha, Chaudhary, et al., 2011) discussed laminar fully developed free-convective flow between two contrary directed moving plates and perceived that increasing Darcy number and dissipation results in a very small increment in skin friction. (Singh & Mathew, 2009a) considered the heat transfer characteristics in three-dimensional MHD flow between two parallel porous plates moving in opposite directions together with transpiration cooling.

1.16.12 FLUID FLOW PAST A STRETCHING SURFACE

The analysis of stretching surfaces still remains very important in the research field. (Sakiadis, 1961) introduced the boundary layer behavior on continuous surfaces, with a constant flow velocity. (Crane, 645–647) extended this, by considering the velocity which is proportional to the distance from the slit. (Grubka & Bobba, 1985) investigated the heat transfer properties of stretching surfaces. The flow and heat transfer properties of electrically conducting fluid past a stretching surface are considered by (Vajravelu & Hadjinicolaou, 1997). The two-dimensional boundary layer flow problem past a stretching surface was developed for the first time by (Crane, 645–647). (Magyari & Keller, 1999b) analytically and numerically described the flow and thermal properties on the exponentially stretching surface. (S. K. Khan & Sanjayanand, 2005) studied the heat transfer characteristics of viscoelastic fluid past an exponentially stretching surface. Naramgari and Sulochana (Naramgari & Sulochana, 2016) performed a study on MHD nanofluid radiative flow with heat generation through an exponentially stretching surface. (Waini, Ishak, & Pop, 2020) explored the hybrid nanofluid flow induced by exponentially stretching and shrinking sheets subject to MHD and radiation effects.

1.16.13 STATISTICAL ANALYSIS

The use of statistical tools for analyzing the effects of numerous physical parameters has intrigued a lot of researchers. A countable number of works, where the ideas of correlation, the slope of linear regression, probable error, regression analysis, sensitivity, and RSM are used to compare and analyze the outcome of various physical quantities. Analysis of physical quantities utilizing statistical techniques is convenient due to its efficiency in producing accurate quantitative results. RSM (Response Surface Methodology) analyses the conjoint impact of effectual parameters (independent variables) on the physical quantity of interest (response or dependent variable). Sensitivity analysis, on the other hand, measures the extent and nature of dependency exhibited by the effectual parameters on the physical quantity of interest. (Fisher, 1950) explained the significance of the correlation coefficient. (Mahanthesh, Shashikumar, Gireesha, & Animasaun, 2019) used the slope of linear regression to characterize the drag coefficient and Nusselt number in his study. (Mackolil & Mahanthesh, 2019a) interpreted the skin friction and heat transfer rate in one of his studies with regression analysis and observed that the heat transfer rate descended with nanoparticle volume fraction and the Dufour number whereas ascending with the radiative heat parameter. (Mackolil & Mahanthesh, 2019b) analyzed heat transfer rate and drag coefficient in a study of Casson nanofluid flow incorporated with radiative heat transfer utilizing RSM. it is observed that the Nusselt number has a positive sensitivity towards thermal radiation, and it is negatively sensitive towards nanoparticle volume fraction and Dufour number.