

Contents

Declaration	i
Certificate	iii
Abstract	vii
Acknowledgements	xiii
List of Figures	xxi
List of Tables	xxvii
1 Introduction	1
1.1 Fluid	1
1.2 Density	1
1.3 Viscosity	1
1.4 Newton's Law of Viscosity	1
1.5 Thermal Conductivity	2
1.6 Types of Fluid	2
1.7 Dilatant Fluid	3
1.8 Pseudoplastic Fluid	3
1.9 Casson Fluid	3
1.10 Carreau Fluid	3
1.11 Laminar Fluid Flow	4
1.12 Steady and Unsteady Fluid Flow	4
1.13 Rotational and Irrotational flow	4
1.14 Mathematical Model	4
1.15 Fundamental Equations of Fluid Flow	4
1.16 Boundary Conditions	5
1.17 Bioconvection	7
1.18 Magnetic Field	8
1.19 Induced Magnetic Field	8

1.20 Dimensional Analysis	8
1.21 Non-Dimensional Parameters	8
1.22 Body Fluid	10
1.23 Nanofluid	10
1.24 Volume Fraction of Nanoparticle	10
1.25 Thermophysical Properties of Nanofluid	10
1.26 Khanafer-Vafai-Lightstone Model	12
1.27 Buongiorno Model	12
1.28 Modified Buongiorno Model	13
1.29 Solution Methodology	13
1.30 Motivation	16
1.31 Objectives	17
1.32 Overview of the Thesis	17
2 Review of Related Literature	23
2.1 Nanofluid	23
2.2 Hybrid Nanofluid	25
2.3 Blood-based Nanofluid	27
2.4 Nanoparticle Shape and Radius	27
2.5 Carreau Nanofluid	28
2.6 Casson Nanofluid	29
2.7 Nanofluid Flow Model	30
2.8 Magnetohydrodynamics	32
2.9 Induced Magnetic Field	32
2.10 Electromagnetohydrodynamics	33
2.11 Bioconvection	34
2.12 Stratification	35
2.13 Flow past a Stretching Sheet	36
2.14 Flow over a Rotating Disk	37
2.15 Stagnation Flow	37
2.16 Slip Boundary Constraint	38
2.17 Heat Source	39
2.18 Passive Control of Nanoparticles	40

2.19	Thermal Radiation	40
2.20	Newtonian Boundary Constraint	41
2.21	Stefan Blowing	41
2.22	Statistical Techniques	42
2.23	Need for the Study	43
3	Bioconvective stagnation-point flow due to induced magnetic field	45
3.1	Introduction	45
3.2	Mathematical Frame	46
3.3	Numerical Frame & Validation	49
3.4	Results & Discussion	50
3.5	Conclusion	63
4	Bioconvective hybrid stagnation-point flow due to induced magnetic field	67
4.1	Introduction	67
4.2	Mathematical Frame	68
4.3	Numerical Frame & Validation	72
4.4	Results & Discussion	73
4.5	Conclusion	83
5	Significance of nanoparticle shape on stagnation-point flow in the presence of induced magnetic field	87
5.1	Introduction	87
5.2	Mathematical Frame	88
5.3	Numerical Frame & Validation	92
5.4	Results & Discussion	93
5.5	Conclusion	108
6	Bioconvective EMHD nanofluid flow past a stretching sheet	113
6.1	Introduction	113
6.2	Mathematical Frame	114
6.3	Numerical Frame & Validation	117
6.4	Results & Discussion	118
6.5	Statistical Frame	128

6.6 Conclusion	132
7 Hydromagnetic Carreau nanoliquid flow over an elongating cylinder	135
7.1 Introduction	135
7.2 Mathematical Frame	136
7.3 Numerical Frame & Validation	139
7.4 Results & Discussion	140
7.5 Statistical Frame	152
7.6 Conclusion	164
8 Bioconvective stagnation-point flow over a rotating stretchable disk	169
8.1 Introduction	169
8.2 Mathematical Frame	170
8.3 Numerical Frame & Validation	174
8.4 Results & Discussion	176
8.5 Statistical Frame	186
8.6 Conclusion	192
9 Nanoliquid flow with irregular heat source and realistic boundary conditions	195
9.1 Introduction	195
9.2 Mathematical Frame	197
9.3 Numerical Frame & Validation	199
9.4 Results & Discussion	201
9.5 Statistical Frame	209
9.6 Conclusion	217
10 Bioconvective magnetized nanomaterial flow subjected to convective thermal heating and Stefan blowing	221
10.1 Introduction	221
10.2 Mathematical Frame	222
10.3 Numerical Frame & Validation	225
10.4 Results & Discussion	227
10.5 Statistical Frame	239

10.6 Conclusion	247
11 Electro-magnetohydrodynamic Casson nanomaterial flow over a nonlinearily stretched surface	251
11.1 Introduction	251
11.2 Mathematical Frame	252
11.3 Numerical Frame & Validation	255
11.4 Results & Discussion	257
11.5 Conclusion	267
12 Conclusion and Future Scope	271
12.1 Concluding Remarks	271
12.2 Future Research Work	274
List of Publications	275
List of Presentations	277
Bibliography	279
Bibliography	279

List of Figures

3.1	Geometrical Frame	46
3.2	$f'(\zeta)$ for differing A values	53
3.3	$f'(\zeta)$ for differing β values	54
3.4	$g'(\zeta)$ for differing A values	54
3.5	$g'(\zeta)$ for differing β values	55
3.6	$g'(\zeta)$ for differing λ values	55
3.7	$\theta(\zeta)$ for differing A values	56
3.8	$\theta(\zeta)$ for differing Ec values	56
3.9	$\theta(\zeta)$ for differing ϕ values	57
3.10	$\theta(\zeta)$ for differing s_1 values	57
3.11	$\psi(\zeta)$ for differing A values	58
3.12	$\psi(\zeta)$ for differing Kr values	58
3.13	$\psi(\zeta)$ for differing s_2 values	59
3.14	$\chi(\zeta)$ for differing A values	59
3.15	$\chi(\zeta)$ for differing s_3 values	60
3.16	Parallel effect of ϕ & β on $Cf_x Re_x^{1/2}$	60
3.17	Parallel effect of ϕ & Ec on $Nu_x Re_x^{-1/2}$	61
3.18	Parallel effect of ϕ & s_1 on $Nu_x Re_x^{-1/2}$	61
3.19	Parallel effect of Kr & s_2 on $Sh_x Re_x^{-1/2}$	62
3.20	Parallel effect of Kr & s_3 on $Nn_x Re_x^{-1/2}$	62
4.1	Geometrical Frame	68
4.2	Variation of A on $f'(\zeta)$ & $g'(\zeta)$	76
4.3	Variation of β on $f'(\zeta)$	76
4.4	Variation of β on $g'(\zeta)$	77
4.5	Variation of λ on $g'(\zeta)$	77
4.6	Parallel effect of β & $\phi_{Fe_3O_4}$ on $Cf_x Re_x^{1/2}$	78

4.7	Parallel effect of β & ϕ_{SWCNT} on $Cf_x Re_x^{1/2}$	78
4.8	Variation of $\phi_{Fe_3O_4}$ & ϕ_{SWCNT} on $\theta(\zeta)$	79
4.9	Variation of S_1 & Ec on $\theta(\zeta)$	79
4.10	Parallel effect of S_1 & $\phi_{Fe_3O_4}$ on $Nu_x Re_x^{-1/2}$	80
4.11	Parallel effect of Ec & ϕ_{SWCNT} on $Nu_x Re_x^{-1/2}$	80
4.12	Variation of Kr , Le , & S_2 on $\psi(\zeta)$	81
4.13	Variation of Pe & S_3 on $\chi(\zeta)$	81
4.14	Parallel effect of (a) S_2 & Le on $Sh_x Re_x^{-1/2}$ and (b) S_3 & Lb on $Nn_x Re_x^{-1/2}$	82
4.15	Parallel effect of (a) S_2 & Kr on $Sh_x Re_x^{-1/2}$ and (b) S_3 & Kr on $Nn_x Re_x^{-1/2}$	82
5.1	Geometrical Frame	88
5.2	$f'(\eta)$ for differing A values	96
5.3	$f'(\eta)$ for differing β values	97
5.4	$f'(\eta)$ for differing ϕ values	97
5.5	$f'(\eta)$ for differing nanoparticle shapes	98
5.6	$Cf_x Re_x^{1/2}$ for differing nanoparticle shapes β values	98
5.7	$Cf_x Re_x^{1/2}$ for differing nanoparticle shapes ϕ values	99
5.8	$g'(\eta)$ for differing A values	99
5.9	$g'(\eta)$ for differing β values	100
5.10	$g'(\eta)$ for differing λ values	100
5.11	$g'(\eta)$ for differing nanoparticle shapes	101
5.12	$\theta(\eta)$ for differing A values	101
5.13	$\theta(\eta)$ for differing ϕ values	102
5.14	$\theta(\eta)$ for differing Q_T values	102
5.15	$\theta(\eta)$ for differing R_d values	103
5.16	$\theta(\eta)$ for differing T_{slip} values	103
5.17	$\theta(\eta)$ for differing nanoparticle shapes	104
5.18	$Nu_x Re_x^{-1/2}$ for differing nanoparticle shapes and ϕ values	104
5.19	$Nu_x Re_x^{-1/2}$ for differing nanoparticle shapes and Q_T values	105
5.20	$Nu_x Re_x^{-1/2}$ for differing nanoparticle shapes and R_d values	105
5.21	$Nu_x Re_x^{-1/2}$ for differing nanoparticle shapes and T_{slip} values	106
5.22	$\psi(\eta)$ for differing A values	106

5.23	$\psi(\eta)$ for differing Le values	107
5.24	$\psi(\eta)$ for differing Kr values	107
5.25	$\psi(\eta)$ for differing C_{slip} values	108
6.1	Geometrical Frame	114
6.2	$f'(\eta)$ for differing M values	120
6.3	$f'(\eta)$ for differing E values	120
6.4	$\theta(\eta)$ for differing ϕ values	121
6.5	$\theta(\eta)$ for differing Ec values	121
6.6	$\theta(\eta)$ for differing s_1 values	122
6.7	$\psi(\eta)$ for differing Kr values	122
6.8	$\psi(\eta)$ for differing s_2 values	123
6.9	$\chi(\eta)$ for differing s_3 values	123
6.10	Variation of $Nn_x Re_x^{-1/2}$ with Kr & Lb	124
6.11	Variation of $Nn_x Re_x^{-1/2}$ with s_3 & Le	124
6.12	Actual versus Estimated $Cf_x Re_x^{1/2}$	131
6.13	Actual versus Estimated $Nu_x Re_x^{-1/2}$	131
7.1	Geometrical Frame	136
7.2	$f'(\eta)$ for differing K values	142
7.3	$f'(\eta)$ for differing We values	142
7.4	$f'(\eta)$ for differing M values	143
7.5	$f'(\eta)$ for differing b_1 values	143
7.6	$\theta(\eta)$ for differing K values	144
7.7	$\theta(\eta)$ for differing M values	144
7.8	$\theta(\eta)$ for differing Rd values	145
7.9	$\theta(\eta)$ for differing b_1 values	145
7.10	$\theta(\eta)$ for differing b_2 values	146
7.11	$\phi(\eta)$ for differing K_r values	146
7.12	$\phi(\eta)$ for differing Nt values	147
7.13	$\phi(\eta)$ for differing Nb values	147
7.14	$\phi(\eta)$ for differing b_3 values	148
7.15	Simultaneous effect of Rd & b_2 on $Nu_x Re_x^{-1/2}$ ($n = 0.7$)	148
7.16	Simultaneous effect of Rd & b_2 on $Nu_x Re_x^{-1/2}$ ($n = 1.4$)	149

7.17	Simultaneous effect of K_r & b_3 on $Sh_x Re_x^{-1/2}$ ($n = 0.7$)	149
7.18	Simultaneous effect of K_r & b_3 on $Sh_x Re_x^{-1/2}$ ($n = 1.4$)	150
7.19	Residual plots for $Nu_{0.7}$	156
7.20	Residual plots for $Nu_{1.4}$	156
7.21	Contour plots for $Nu_{0.7}$	157
7.22	Surface plots for $Nu_{0.7}$	158
7.23	Contour plots for $Nu_{1.4}$	159
7.24	Surface plots for $Nu_{1.4}$	160
7.25	Bar charts showcasing the sensitivity of $Nu_{0.7}$	161
7.26	Bar charts showcasing the sensitivity of $Nu_{1.4}$	162
8.1	Geometrical Frame	170
8.2	Impression of S on $F(\eta)$	177
8.3	Impression of S on $G(\eta)$	178
8.4	Impression of M on $F(\eta)$	178
8.5	Impression of M on $G(\eta)$	179
8.6	Impression of ω on $G(\eta)$	179
8.7	Impression of ϕ on $\theta(\eta)$	180
8.8	Impression of S_1 on $\theta(\eta)$	180
8.9	Impression of Kr on $\psi(\eta)$	181
8.10	Impression of S_2 on $\psi(\eta)$	181
8.11	Impression of S_3 on $\chi(\eta)$	182
8.12	Simultaneous impression of ω and M on $C_f Re^{1/2}$	182
8.13	Simultaneous impression of ϕ and S on $C_f Re^{1/2}$	183
8.14	Simultaneous impression of S_1 and M on $Nu_r Re^{-1/2}$	183
8.15	Simultaneous impression of ϕ and S on $Nu_r Re^{-1/2}$	184
8.16	Simultaneous impression of S_2 and Kr on $Sh_r Re^{-1/2}$	184
8.17	Simultaneous impression of S_2 and Kr on $Nn_r Re^{-1/2}$	185
8.18	Simultaneous impression of S_3 and Lb on $Nn_r Re^{-1/2}$	185
8.19	Residual versus Predicted value	188
8.20	Surface plots for $Nu_r Re^{-1/2}$	191
9.1	Geometrical Frame	196
9.2	$f'(\zeta)$ for distinct values of n	202

9.3	$f'(\zeta)$ for distinct values of M	202
9.4	$\theta(\zeta)$ for distinct values of n	203
9.5	$\theta(\zeta)$ for distinct values of M	203
9.6	$\theta(\zeta)$ for distinct values of R_d	204
9.7	$\theta(\zeta)$ for distinct values of Q_E	204
9.8	$\theta(\zeta)$ for distinct values of Nt	205
9.9	$\psi(\zeta)$ for distinct values of n	205
9.10	$\psi(\zeta)$ for distinct values of Nt	206
9.11	$\psi(\zeta)$ for distinct values of Nb	206
9.12	Simultaneous effect of M and ϕ on $Cf_x Re_x^{1/2}$	207
9.13	Simultaneous effect of M and ϕ on $Nu_x Re_x^{-1/2}$	207
9.14	Simultaneous effect of R_d and Q_E on $Nu_x Re_x^{-1/2}$	208
9.15	Residual plots	212
9.16	Contour plots for on $Nu_x Re_x^{-1/2}$	213
9.17	Surface plots for on $Nu_x Re_x^{-1/2}$	214
9.18	Bar charts showcasing the sensitivity of on $Nu_x Re_x^{-1/2}$	215
10.1	Geometrical Frame	222
10.2	$f'(\xi)$ for distinct n values	228
10.3	$f'(\xi)$ for distinct M values	229
10.4	$\theta(\xi)$ for distinct n values	229
10.5	$\theta(\xi)$ for distinct M values	230
10.6	$\theta(\xi)$ for distinct Q_T values	230
10.7	$\theta(\xi)$ for distinct Q_E values	231
10.8	$\theta(\xi)$ for distinct Rd values	231
10.9	$\theta(\xi)$ for distinct Bi values	232
10.10	$\psi(\xi)$ for distinct n values	232
10.11	$\psi(\xi)$ for distinct Kr values	233
10.12	$\psi(\xi)$ for distinct Le values	233
10.13	$\chi(\xi)$ for distinct Pe values	234
10.14	$\chi(\xi)$ for distinct Ω values	234
10.15	$\chi(\xi)$ for distinct Lb values	235
10.16	Simultaneous impact of M and n on $Cf_x Re_x^{1/2}$	235
10.17	Simultaneous impact of M and n on $Nu_x Re_x^{-1/2}$	236

10.18	Simultaneous impact of Q_T and Q_E on $Nu_x Re_x^{-1/2}$	236
10.19	Simultaneous impact of Rd and Bi on $Nu_x Re_x^{-1/2}$	237
10.20	Simultaneous impact of Lb and Kr on $Nn_x Re_x^{-1/2}$	237
10.21	Simultaneous impact of Pe and Ω on $Nn_x Re_x^{-1/2}$	238
10.22	Residual plots	242
10.23	Contour plots for on $Nu_x Re_x^{-1/2}$	243
10.24	Surface plots for on $Nu_x Re_x^{-1/2}$	244
10.25	Bar charts showcasing the sensitivity of on $Nu_x Re_x^{-1/2}$	245
11.1	Geometrical Frame	252
11.2	Deviations in $f'(\eta)$ with m	257
11.3	Deviations in $f'(\eta)$ with M	258
11.4	Deviations in $f'(\eta)$ with E	258
11.5	Deviations in $f'(\eta)$ with β	259
11.6	Deviations in $f'(\eta)$ with ϕ	259
11.7	Deviations in $f'(\eta)$ with h	260
11.8	Deviations in $f'(\eta)$ with R_{np}	260
11.9	Deviations in $f'(\eta)$ with L_1	261
11.10	Deviations in $f'(\eta)$ with L_2	261
11.11	Deviations in $\theta(\eta)$ with M	262
11.12	Deviations in $\theta(\eta)$ with ϕ	262
11.13	Deviations in $\theta(\eta)$ with h	263
11.14	Deviations in $\theta(\eta)$ with R_{np}	263
11.15	Deviations in $\theta(\eta)$ with L_3	264
11.16	Deviations in $\theta(\eta)$ with A^*	264
11.17	Deviations in $Cf_x Re_x^{1/2}$ with differing parameters	265
11.18	Deviations in $Nu_x Re_x^{-1/2}$ with differing parameters	265

List of Tables

3.1 Comparison of drag coefficient ($Cf_x Re_x^{1/2}$) for different A values between the present study and the works of Iqbal, Azhar, et al., 2017 and Hayat et al., 2015, 2016 when $\phi = \beta = 0$	51
3.2 Comparison of $Nu_x Re_x^{-1/2}$ and $-Cf_x Re_x^{1/2}$ for differing ϕ, β, λ values between the present study and (Iqbal, Azhar, et al., 2017)	51
3.3 Thermophysical properties of water and SWCNT	51
4.1 Comparison of drag coefficient ($Cf_x Re_x^{1/2}$) with Hayat et al., 2015, 2016; Iqbal, Azhar, et al., 2017 for different A values when $\phi_{Fe_3O_4} = \phi_{SWCNT} = \beta = \lambda = 0$	73
4.2 Thermophysical properties of water, Fe_3O_4 and $SWCNT$	74
5.1 Comparison of drag coefficient ($Cf_x Re_x^{1/2}$) for different A values between the present study and the works of Iqbal, Azhar, et al., 2017 and Hayat et al., 2015, 2016 when $\phi = \beta = 0$	94
5.2 Thermophysical properties of blood and silver	94
5.3 Variation in $Sh_x Re_x^{-1/2}$ when $A = 0.5$, $\beta = 0.1$, $\lambda = 0.5$ & $\phi = 0.1$	94
6.1 Comparison of $Nu_x(Re_x)^{-1/2}$ for differing Pr values between the present study and the works of W. A. Khan et al., 2010; Wang, 1989 when $M = E = Kr = Ec = Le = Lb = Pe = \Omega = s_1 = s_2 = s_3 = 0$	118
6.2 Thermophysical properties of water and SWCNT	118
6.3 Variation in $Cf_x Re_x^{1/2}$ when $Kr = 1$, $Ec = Le = Lb = 0.3$, $Pe = \Omega = 0.5$ & $s_1 = s_2 = s_3 = 0.1$	125
6.4 Variation in $Nu_x Re_x^{-1/2}$ when $Kr = 1$, $Le = Lb = 0.3$, $Pe = \Omega = 0.5$ & $s_2 = s_3 = 0.1$	126
6.5 Variation in $Sh_x Re_x^{-1/2}$ when $M = 1$, $Ec = Lb = 0.3$, $Pe = \Omega = 0.5$ & $s_1 = s_3 = 0.1$	127

6.6	Correlation Coefficient (r), Probable Error (PE) and $\left \frac{r}{PE} \right $ of drag coefficient	129
6.7	Correlation Coefficient (r), Probable Error (PE) and $\left \frac{r}{PE} \right $ of heat transfer rate	129
7.1	Comparison of $Nu_x Re_x^{-1/2}$ for different Pr values with the work of W. A. Khan et al., 2010 when $Nb \rightarrow 0$ and $M = K = We = Sc = Nt = Rd = Kr = b_1 = b_2 = b_3 = 0$	140
7.2	Variation in $\frac{1}{2}Cf_x Re_x^{1/2}$ for various parameters when $Pr = 4$ and $Sc = 1.2$	151
7.3	Effectual parameter levels	153
7.4	CCD experimental design and the corresponding responses	154
7.5	ANOVA table ($Nu_{0.7}$)	155
7.6	ANOVA table ($Nu_{1.4}$)	155
7.7	Sensitivity of $Nu_{0.7}$ & $Nu_{1.4}$ when $A = 0$	164
8.1	Resemblance of $F'(0)$ and $-G'(0)$ with the work of Mustafa et al., 2016 when $S = \phi = Nt = Le = Lb = \Omega = Pe = Kr = S_1 = S_2 = S_3 = 0$, $Nb \rightarrow 0$, and $Pr = 1$	175
8.2	Thermophysical properties of water and magnetite nanoparticles	176
8.3	Effectual parameter levels	187
8.4	Four-factor CCD experimental design and the corresponding responses	189
8.5	ANOVA table	190
9.1	Comparison of $Nu_x Re_x^{-1/2}$ for different values of Pr_f with the work of W. A. Khan et al., 2010 when $n = 1$. $Nb \rightarrow 0$ and $M = Nt = \phi = R_d = Q_E = Le = 0$	200
9.2	Thermophysical properties of water and titanium dioxide	201
9.3	Effectual parameter levels	210
9.4	Experimental design with actual and estimated response values	211
9.5	ANOVA table and regression coefficients	212
9.6	Sensitivity of $Nu_x Re_x^{-1/2}$ when $A = 0$	217
10.1	Resemblance of $-\theta'(0)$ for ordinary fluid when $Pr_f = 1$	226
10.2	Effectual parameter levels	239

10.3 Experimental design with actual and estimated response values	241
10.4 ANOVA table and regression coefficients	242
10.5 Sensitivity of $Nu_x Re_x^{-1/2}$ when $A = 0$	246
11.1 Resemblance of $-\theta'(0)$ for ordinary fluid when $Pr_f = 1$	256
11.2 Thermophysical properties of blood and gold	256

