ON JOINT IMPORTANCE MEASURES FOR MULTISTATE RELIABILITY SYSTEMS

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Abstract

The use of importance and joint importance measures to identify the weak areas of a system and signify the roles of components in either causing or contributing to proper functioning of the system, is explained by several researchers in system engineering. But a few research outputs are available in literature for finding joint importance measures for two or more components. This paper introduces, new Joint Reliability Achievement Worth (JRAW), Joint Reliability Reduction Worth (JRRW) and Joint Reliability Fussell-Vesely measure (JRFV) for two components, of a multistate system. This is a new approach to find out the joint effect of group of components in improving system reliability. A steady state performance level distribution with restriction to the component's states is used to evaluate the proposed measures. Universal generating function (UGF) technique is applied for the evaluation of proposed joint importance measures. An illustrative example is provided

Keywords: Multistate system, reliability, joint importance measure, universal generating function.

I. Introduction

Importance and joint importance measures provides useful information to understand the system and apply reliability improvement activities. There are several importance measures available in literature, [1], [2], [3]. Interaction importance of groups of components, with respect to output performance measure(OPM)s, reliability and expected output performance is more helpful to the designers, engineers and managers to arrive at a decision, [4].

The joint importance measures of components for MSS with respect to various OPMs like reliability and expected output performance with reference to the existing measures of importance are discussed in literature in the Birnbaum sense, [5]. Research on joint importance measures for multistate systems is very useful for the researchers, [9]. But, measuring the role of interaction of components in a group consisting two components, in performance measure achievement, reduction and fractional contribution sense, is an unexplored one. In this paper, for two components of binary and MSS, Joint Reliability Achievement Worth (JRAW), Joint Reliability Reduction Worth (JRRW), and Joint Reliability Fussel-Vesely (JRFV) importance measures are introduced by considering groups with two components. JRAW measures the reliability achievement when interaction effect of two components changes from lower level to higher level, JRRW measures the reliability reduction of system when interaction effect of two components changes from higher level to lower level and JRFV measures the fractional contribution of interaction effect of two components in improving reliability of system. These measures are generalized to the expected output performance.

A steady state performance level distribution for the system is considered for obtaining the proposed measures, [6]. The information derived by these joint importance measures allows the analyst to judge, based on their interaction effect of two components for system OPM improvement: how to give reliability operations?.

Let the components i and j are restricted with respect to performance thresholds α , and β respectively. Let $OPM_{i,j}^{\leq\alpha,\leq\beta}$, $OPM_{i,j}^{>\alpha,\leq\beta}$, $OPM_{i,j}^{\leq\alpha,>\beta}$ and $OPM_{i,j}^{>\alpha,>\beta}$ are state space restricted OPMs. If the performance measure of series system is sum performance measure of components, UGF method is found to be useful to evaluate system performance. Power generation, oil transportation systems etc are such systems.

The paper is arranged as follows. The performance measures of the MSS and new joint importance measures of two components of the binary and MSS are introduced in section II. Discussion is given in section III. Illustrative example is given in section IV. Conclusion is given in section V.

II. New Joint Importance Measures

The performance measures used for the present study are discussed below. Using the performance measure Reliability and expected output performance measure, the new joint importance measures are introduced.

I. Performance Measures of a Multistate system

A multistate system with multistate components is considered. Let the structure function of a MSS at time t be denoted by $\varphi\big(X(t)\big) = i$, $i \in \{0,1,2,\dots,M\}$, where $X(t) = (X_1(t), X_2(t), \dots, X_n(t))$, $X_i(t) \in \{0,1,2,\dots,M_i\}$, and $M = \max_{1\leq i\leq n}\{M_i\}$. Let the output performance of the MSS at time t, $W(t)$, where $W(t) \in \{w_i, i = 0,1,\dots,M\}$ corresponds to the system state $\varphi(X(t)) = i$. Let

$$p_i = \lim_{t\to\infty} \Pr\{W(t) = w_i\} = \lim_{t\to\infty} \Pr\{\varphi(X(t)) = i\},\ 0\leq i \leq M.$$

Then the steady state performance distribution of the output performance of system, **w**={w_i, $0\leq i \leq M$} is represented by **p**={p_i, $0\leq i \leq M$}. Steady state expected performance is

$$E(W) = \textstyle\sum_{i=0}^{M} p_i w_i. \tag{1}$$

and expected system state is

$$E_s(\varphi(X)) = \textstyle\sum_{i=0}^{M} i p_i. \tag{2}$$

For constant demand D_k, to state k of the multistate system, reliability is

$$R(t) = Pr\{W(t) \geq D_k\} = Pr\{\varphi(t) \geq k\}. \tag{3}$$

The stationary reliability is

$$R(D_k) = \sum_{i=0}^{M} p_i 1(w_i - D_k). \tag{4}$$

These performance measures are commonly used for reliability importance analysis, [6].

II. New Joint Importance Measures for two components in the MSS

Suppose now the components are statistically independent and reliabilities are known. In order to understand the interaction effect of two components in reliability achievement, reliability reduction and fractional contribution to reliability improvement, three joint importance measures are proposed.

Joint Reliability Achievement Worth (JRAW)

Reliability achievement worth is a measure to understand the improvement in system reliability. Consider a group of two components, with reference to the interaction, the groups having highest reliability achievement worth will be most important to improve the existing level of reliability. In order to assess the change in reliability by the presence or functioning or switching to functioning states of a group, the Joint Reliability Achievement Worth (JRAW) has to be measured.
The role of interaction of components in a group consisting *2* components, in increasing reliability of system, define the following:
${ci_j}^+$, indicate i_jth component is in functioning states or up states
${ci_j}^-$: indicate i_jth component is in unreliable states or down states
$I_{12} = (c1^+ - c1^-)(c2^+ - c2^-) = (c1^+ - c1^-)c2^+ - (c1^+ - c1^-)c2^- = I_{12}^+ - I_{12}^-$, the contrast of interaction of the component 1 and 2, while they switch from reliable states to down states, where $I_{12}^+ = (c1^+ - c1^-)c2^+$ is the high level interaction contrast of component 1 and 2 and $I_{12}^- = (c1^+ - c1^-)c2^-$ is the low level interaction contrast of component 1 and 2.
Let $\partial R_\varphi(i) = P(\varphi(X(t)) = 1, {I_i}^+) - P(\varphi(X(t)) = 1, {I_i}^-) = P(\varphi(X(t)) = 1, X_i(t) = 1) - P(\varphi(X(t)) = 1, X_i(t) = 0)$ i=1,2,…,n, the Birnbaum importance of component i, and
$\partial R_\varphi(i,j) = \partial R(\varphi(X(t)) = 1, {I_{ij}}^+) - \partial R(\varphi(X(t)) = 1, {I_{ij}}^-) = [P(\varphi(X(t)) = 1, X_i(t) = 1, X_j(t) = 1) - P(\varphi(X(t)) = 1, X_i(t) = 0, X_j(t) = 1)] - [P(\varphi(X(t)) = 1, X_i(t) = 1, X_j(t) = 0) - P(\varphi(X(t)) = 1, X_i(t) = 0, X_j(t) = 0)]$, joint Birnbaum joint importance of components *i* and *j.*

Now define JRAW of two components.
Let
$R_{\{i+,j+\}} = P(\varphi(X(t)) = 1, X_i(t) = 1, X_j(t) = 1)$,
$R_{\{i-,j+\}} = P(\varphi(X(t)) = 1, X_i(t) = 0, X_j(t) = 1)$,
$R_{\{i+,j-\}} = P(\varphi(X(t)) = 1, X_i(t) = 1, X_j(t) = 0), and$
$R_{\{i-,j-\}} = P(\varphi(X(t)) = 1, X_i(t) = 0, X_j(t) = 0)$.

$$JRAW = \frac{Maximum\ Reliability\ due\ to\ high\ level\ interaction\ effect\ of\ two\ components}{The\ present\ reliability\ level.}$$

$$JRAW_{i,j} = \frac{[R_{\{i+,j+\}} - R_{\{i-,j+\}}]}{R}$$

The $JRAW_{i,j}$ measure quantifies the maximum possible achievement of reliability due to interaction effect of component *i* and, *j* which switches from lower level to higher level. For *i*th multistate component with performance threshold α, let $k_{i\alpha}$ be the state in the ordered set of states of component *i* such that $x_{ik_{i\alpha}} \leq \alpha < x_{ik_{i\alpha}+1}$, [6]. For a constant demand D_k, to define Multistate Joint

Reliability Achievement Worth (MJRAW) of components i and j, let,

$R_{\{i\geq\alpha,j\geq\beta\}} = P\left(\varphi(X(t)) \geq k, X_i(t) \geq x_{ik_{i\alpha}}, X_j(t) \geq x_{jk_{j\beta}}\right)$,

$R_{\{i<\alpha,j\geq\beta\}} = P\left(\varphi(X(t)) \geq k, X_i(t) < x_{ik_{i\alpha}}, X_j(t) \geq x_{jk_{j\beta}}\right)$,

$R_{\{i\geq\alpha,j<\beta\}} = P\left(\varphi(X(t)) \geq k, X_i(t) \geq x_{ik_{i\alpha}}, X_j(t) < x_{jk_{j\beta}}\right)$,

and

$R_{\{i<\alpha,j<\beta\}} = P\left(\varphi(X(t)) \geq k, X_i(t) < x_{ik_{i\alpha}}, X_j(t) < x_{jk_{j\beta}}\right)$

where α is the performance threshold and $x_{ik_{i\alpha}}$ performance of component i in state $k_{i\alpha}$, β is the performance threshold and $x_{jk_{j\beta}}$is the performance of component *j* in the state $k_{j\beta}$, *i, j, =1,2,…,n*. Thus, MJRAW of two components *i* and *j* can be defined as,

$$MJRAW_{i,j} = \frac{\left[R_{\{i\geq,j\geq\}} - R_{\{i<,j\geq\}}\right]}{R}$$

MJRAW measures the reliability achievement worth of interaction effect of two components.

Joint Reliability Reduction Worth (JRRW)

To measure the role of interaction effect of two components in reducing the present reliability, Joint Reliability Reduction Worth (JRRW) is introduced in this section. To examine how the decrease in reliability happens by the decreased level or low level of interaction effect of two components, JRRW can be defined as follows.

Let

$$R_G^- = The\ decreased\ reliability\ level\ by\ the\ low\ level\ \ interaction\ of\ two\ omponents$$

and $R_0 = Present\ reliability\ level.$ The JRRW of a module is defined as:

$$JRRW = \frac{R_0}{R_G^-}$$

JRRW of two binary components *i* and *j* is

$$JRRW = \frac{Present\ Reliability\ Level}{Reliability\ when\ interaction\ of\ two\ components\ is\ at\ low\ level}$$

$$JRRW_{i,j} = \frac{R}{\left[R_{\{i+,j-\}} - R_{\{i-,j-\}}\right]}$$

The $JRRW_{i,j}$ measure of two components *i* and *j,* quantifies the maximum possible reduction of reliability due to low level of interaction effect of component *i* and *j*. For a constant demand D_k, Multistate Joint Reliability Reduction Worth (MJRRW) of a module consisting of two components *i* and *j* is defined as,

$$MJRRW_{i,j} = \frac{R}{[R_{\{i\geq\alpha,j<\beta\}} - R_{\{i<\alpha,j<\beta\}}]}$$

MJRRW measures the reliability reduction worth of interaction effect of two components *i and j.*

Joint Reliability Fussel-Vesely (JRFV) Measure

To measure the fractional contribution of interaction effect of components to the increase of reliability, Joint Reliability Fussel-Vesly (JRFV) measure can be defined. JRFV measure can be expressed as, $JRFV = \frac{\mathrm{R_0 - R_G^-}}{\mathrm{R_0}}$.

$$JRFV = \frac{Present\ Reliability\ Level - Reliability\ when\ interaction\ of\ two\ components\ is\ in\ low\ level}{Present\ Reliability\ Level}$$

$$JRFV_{i,j} = \frac{R - \left[R_{\{i+,j-\}} - R_{\{i-,j-\}}\right]}{R}$$

The $JRFV_{i,j}$ measure of two components *i* and *j,* quantifies the maximum fractional contribution of reliability due to high level of interaction effect of component *i* and *j*. For a constant demand D_k, Multistate Joint Fussel-Vesly (MJRFV) of two components i and j is defined as,

$$MJRFV_{i,j} = \frac{R - [R_{\{i \geq \alpha, j < \beta\}} - R_{\{i < \alpha, j < \beta\}}]}{R}.$$

MJRFV measures the reliability FV of a module consisting of two components.

For the expected output performance measure, define Multistate Joint Output Performance Measure Achievement Worth (MJOPMAW), Multistate Joint Output Performance Measure Reduction Worth (MJOPMRW) and Multistate Joint Output Performance Measure Fussel-Vesely (MJOPFV) measures as below. For two components i and j,

$$MJOPMAW_{i,j} = \frac{[OPM_{\{i\geq, j\geq\}} - OPM_{\{i<, j\geq\}}]}{OPM}$$

$$MJOPMRW_{i,j} = \frac{OPM}{[OPM_{\{i \geq \alpha, j < \beta\}} - OPM_{\{i < \alpha, j < \beta\}}]}$$

$$MJOPMFV_{i,j} = \frac{OPM - [OPM_{\{i \geq \alpha, j < \beta\}} - OPM_{\{i < \alpha, j < \beta\}}]}{OPM}$$

A component's performance restriction approach can be adopted for the computation of the joint importance measures and UGF method can be adopted for the evaluation procedure, [6], [7]. The coefficients of UGFs are used for the evaluation of joint importance measures, [8].

III. Discussion

In binary and multistate context, the proposed measures quantify the RAW, RRW and FV measures of interaction effect of two components. Many of the complex systems are made up of two or more components. MJRAW measures the reliability achievement when interaction effect of two components changes from lower level to higher level, MJRRW measures the reliability reduction of system when interaction effect of two components changes from higher level to lower level and MJRFV measures the fractional contribution of interaction effect of two components. Using the information of MJRAW, it is easy to understand and identify the pair of components with highest contribution to system reliability improvement. MJRRW provides the information regarding the group which induce lowest reduction in system reliability with lower level of group performance. The fractional contribution in reliability improvement of a pair of components can be measured using MJRFV. MJOPMAW, MJOPMRW and MJOMPFV measures are useful when a researcher uses output performance measure, expected output performance measure.

IV. Illustrative Example

Consider a system made up of n = 3 multi-state components in series logic. Component states are 0, 1, 2, 3 and 4, with corresponding values of performance x_{j0}=0, x_{j1}=25, x_{j2}=50, x_{j3}=75, x_{j4}=100, j=1, 2, 3,4 (see Figure 1).

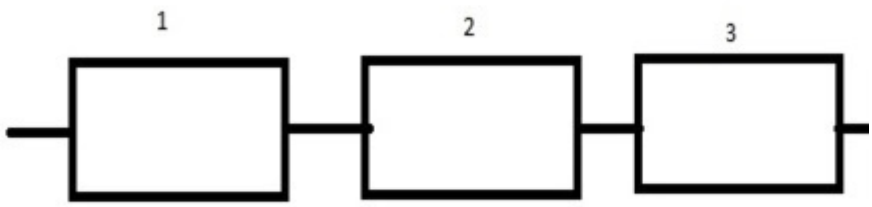


Figure 1: Series system

The probability distribution of component j in state k, p_{jk}, is given in Table 1. Let 0, 1 and 2 are unreliable states for $< \alpha$ or $< \beta$ and 3 and 4 ate reliable states for $\geq \alpha$ or $\geq \beta$.

Table 1. Probability distributions of components 1, 2 and 3.

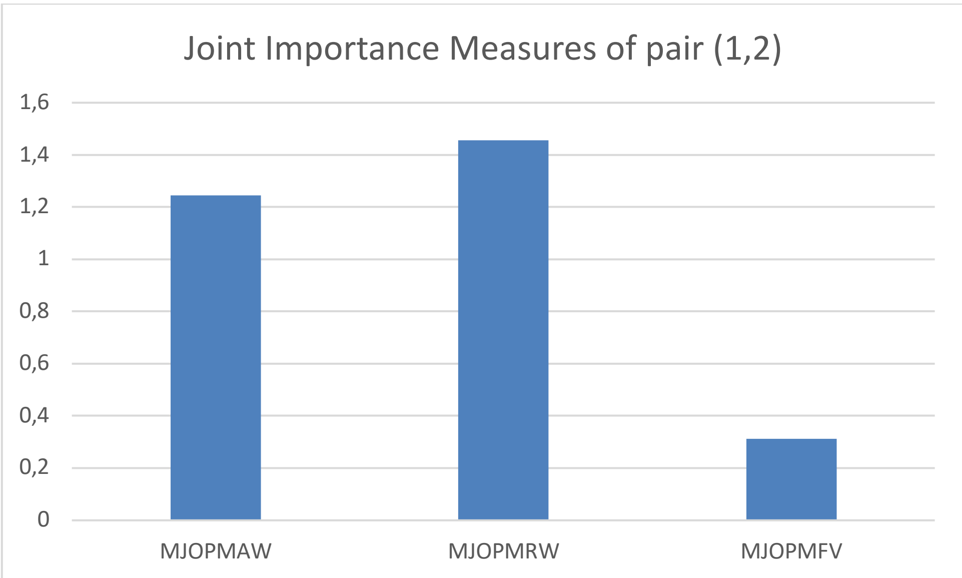
Probability Distribution	1		2		3	
P(Xi0=0)	p10	=0.1	p20	=0.15	p30	=0.4
P(Xi1=25)	p11	=0.1	p21	=0.2	p31	=0.1
P(Xi2=50)	p12	=0.5	p22	=0.3	p32	=0
P(Xi3=75)	p13	=0.2	p23	=0.2	p33	=0.1
P(Xi4=100)	p14	=0.1	p24	=0.15	p34	=0.4

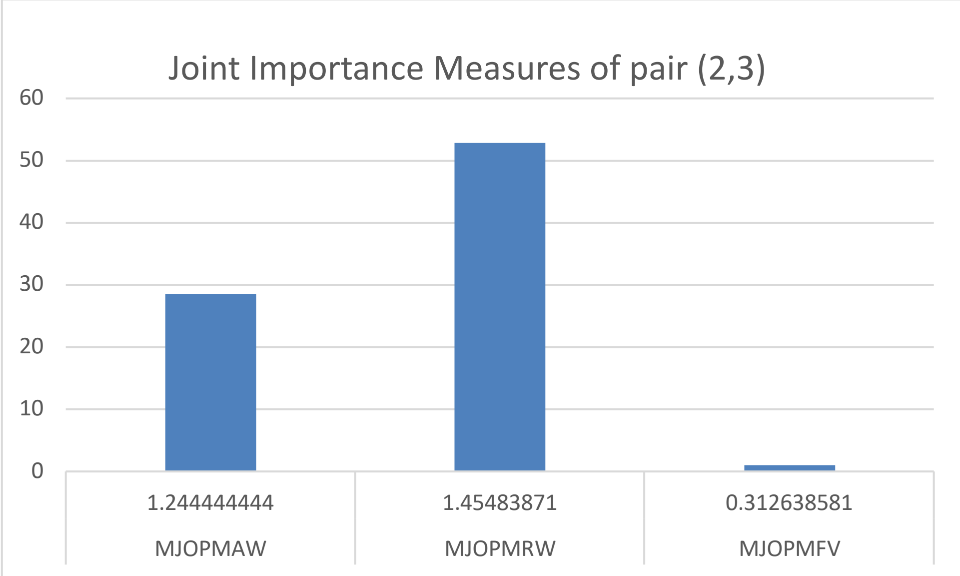
Table 2. Multistate joint importance measures

For components 1, 2	For components 2, 3
MJOPMAW= 1.244444444	MJOPMAW=28.52525253
MJOPMRW= 1.45483871	MJOPMRW= 52.8
MJOPMFV =0.312638581	MJOPMFV =0.981060606

Multistate joint importance measures are given in Table 2 and plotted in Figure 2. The sign and size of the value of joint importance measure with regard to their impact on expected system output performance are found to be different. So, a numerical comparison can be made.

Consider two groups, Group 1 with components 1 and 2 and Group 2 with components 2, and 3. Highest values for MJOPMAW, MJOPMRW and MJOPMFV are attained for pair of components 2 &3. Highest values of joint importance measures are due to highest influence of those groups in change of system reliability.





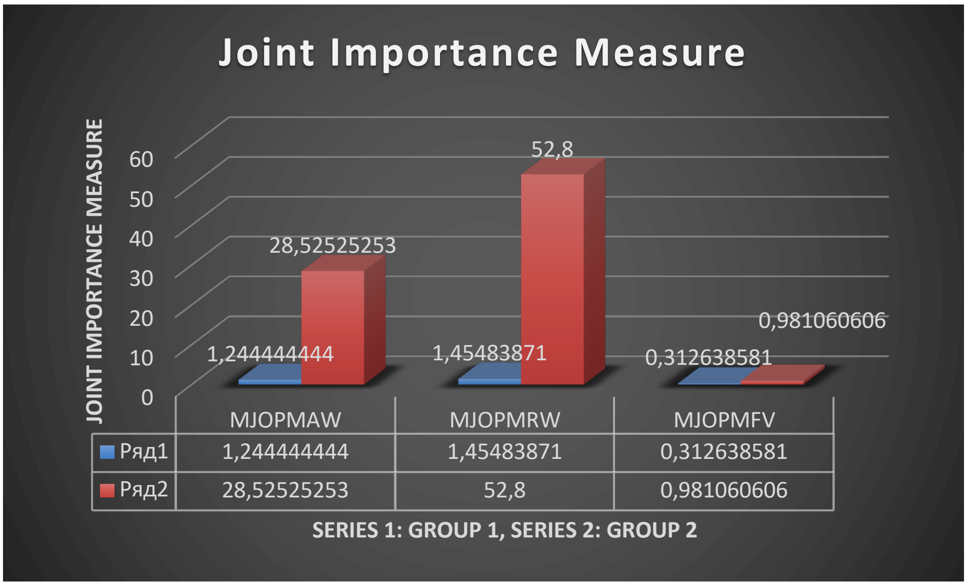


Figure 2: Multistate joint importance measures of Group 1 and Group 2

This information can be used to provide more reliability operations for different pair of components. Highest values in various importance measures indicates the need of highest care in reliability operations. To understand the dynamics of system reliability change, one can use the proposed importance measures.

V. Conclusion

This paper introduced three module joint importance measures for MSSs with reference to the OPMs reliability and expected system output performance. The joint importance measures MJRAW, MJRRW, and MJRFV for two components are introduced and generalized to expected output performance measure. The new joint importance measures are useful for giving priority for reliability improvement activities. The UGF method is used to evaluate the joint importance measures, in which the system performance is measured in terms of productivity or capacity. Joint importance measure values provide useful information for reliability improvement activities. The value and size of the importance measure can be used to make a comparison between different groups.

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