RT&A, No 4 (76)
Volume 18, December 2023

ESTIMATION OF STRESS STRENGTH RELIABILITY USING PRANAV DISTRIBUTION

Ankitha Lukose, Chacko V M

•

Department of Statistics
St.Thomas College(Autonomous)
Thrissur, Kerala, 680 001, India
ankithalukose17@gmail.com
chackovm@gmail.com

Abstract

This paper deals with the estimation of stress strength reliability parameter R, which is the probability of Y less than X when X and Y are two independent distribution with different scale parameter and same shape parameter. The maximum likelihood method is used to find an estimator for R. We also obtain the asymptotic distribution of the maximum likelihood estimator of R. Based on this asymptotic distribution, the asymptotic confidence interval can be obtained. We also propose bootstrap confidence interval for the parameter R. Analysis of a simulated data and a real life data have been presented for illustrative purposes.

Keywords: stress strength model, maximum-likelihood estimator, bootstrap confidence intervals, asymptotic distributions and confidence interval

1. Introduction

One of the significant, challenging, but manageable problems in reliability analysis is the calculation of stress-strength reliability using a variety of distributions. Estimating the stress-strength parameter, R, is very helpful in the statistical literature. For instance, if X represents the strength of a system under a stress, Y, then R is a measure of system performance that arises naturally from the mechanical dependability of a system. Only when the applied stress exceeds the system's strength at any point does the system fail. Many lifespan distributions are utilised in reliability analysis. Terms like exponential, Weibull, log-Normal, and their generalisations are commonly used in dependability analysis. The exponential, Lindley, and Weibull distributions are more often used than the gamma and lognormal distributions because their survival functions can both be expressed in closed forms and do not require numerical integration. While sharing a common parameter, the exponential and Lindley distributions differ in that the hazard rate of the exponential is constant whereas the hazard rate of the Lindley is monotonically dropping. Although the Lindley distribution has been used by many academics to model lifetime data and is crucial for understanding stress-strength reliability modelling, there are numerous instances in which modelling actual lifetime data may not be appropriate from a theoretical or practical standpoint. Recently, a number of academics presented many distributions, with the new ones showing a better fit than the currently popular distributions. In order to fix a problem with new models while utilising better fitted models in stress-strength analysis, one may need to carefully examine the estimation process. If the estimation process fails using the available methodologies, one may not be able to do so. Therefore, it is crucial to estimate multiple reliability factors, and researchers must focus more on estimates while utilising better fitted models.

In the literature, it has been debated how to estimate a stress strength model's reliability or survival probability when X and Y have known distributions. A number of authors have examined the survival probabilities of a single component stress-strength (SSS) model for various distributions, including Raqab and Kundu[12], Kundu and Gupta [9], [6], Constantine and Karson[4], and Downtown[5]. The issue of estimating R has been investigated by a number of authors. Church and Harris [3] developed the MLE of R when X and Y are independently and normally distributed. Awad et al study's on the MLE of R under the condition that X and Y have bivariate exponential distributions was published in 1981. In a simulation research, Awad and Gharraf [2] compared three estimations of R when X and Y are independent Burr random variables with different distributions. Estimates for R where X and Y are Burr Type X distributions were reported by Ahmad et al.[1] and Surles and Padgett[16],[17]. Other papers which describes the same idea on different distributions are Akhila and Chacko [18],Kundu and Raqab [11],saraccouglu,Kinaci and Kundu [14] and Shahsanaei, Fatemeh and Daneshkhah, Alireza [15].

In this paper, we consider the problem of estimating the stress strength parameter $R = P[Y < X]$, when X and Y be independent strength and stress random variables having Pranav distribution with parameters θ_1 and θ_2 respectively. Krishna Kumar Shukla [8]introduced Pranav distribution which is a mixture of two distributions, exponential distribution having scale parameter θ and gamma distribution having shape parameter 4 and scale parameter θ, and their mixing proportion of θ, $\frac{\theta^4}{\theta^4+6}$ and $\frac{6}{\theta^4+6}$ respectively. The probability density function (pdf) of Pranav lifetime distribution can be defined as

$$f(x,\theta) = \frac{\theta^4}{\theta^4+6}(\theta + x^3)e^{-\theta x},\ x > 0, \theta > 0 \tag{1}$$

The corresponding cumulative distribution function(c.d.f) is

$$F(x,\theta) = 1 - \left[1 + \frac{\theta x(\theta^2 x^2 + 3\theta x + 6}{\theta^4 + 6}\right] e^{-\theta x},\ x > 0, \theta > 0 \tag{2}$$

The estimation of the stress strength parameter $R = P[Y < X]$, when X and Y are both one-parameter Pranav distributions with parameter θ_1 and θ_2 respectively, is an unsolved problem. Statistical inference on stress-strength parameters are important in reliability analysis. It is observed that the maximum likelihood estimators can be obtained implicitly by solving two nonlinear equations, but they cannot be obtained in closed form. So, MLE' s of parameters are derived numerically. It is not possible to compute the exact distributions of the maximum likelihood estimators, and we used the asymptotic distribution and we constructed approximate confidence intervals of the unknown parameters.

The rest of the paper is organized as follows. In Section 2, the MLE of R is computed. The asymptotic distribution of the MLE' s are provided in Section 3. Bootstrap confidence interval is presented in Section 4. In Section 5, simulation study is given. Theoretical results are verified by analyzing data set in Section 6 and conclusions are given in Section 7.

2. Maximum Likelihood Estimator of R

In this section, the procedure of estimating the reliability of $R = P[Y < X]$ models using Pranav distributions, is considered. It is clear that

$$\begin{aligned} R &= P[Y < X] \\ &= \int_0^\infty f(x,\theta_1)F(x,\theta_2)\,dx \end{aligned} \tag{3}$$

where f (x,y), is the joint probability density function (pdf) of random variables X and Y, having Pranav distributions. If the r.v' s X and Y are independent, then $f(x,y) = f(x)g(y)$, where f (x) and g(y) are the marginal pdf' s of X and Y, so that

$$R = \int_0^\infty \left(\frac{\theta^4}{\theta^4+6}(\theta + x^3)e^{-\theta x} \right) \left(1 - \left[1 + \frac{\theta x(\theta^2 x^2 + 3\theta x + 6}{\theta^4+6} \right] e^{-\theta x} \right) dx \tag{4}$$

On simplification we get

$$R = 1 - \frac{\theta_1 \begin{bmatrix} 360\theta_1\theta_2^2 + 1080\theta_2^3 + 144\theta_2(\theta_1+\theta_2)^2 + \\ 6(\theta_1\theta_2^3 + \theta_2^4 + 6)(\theta_1+\theta_2)^3 + \\ 6(\theta_1\theta_2)^2(\theta_1+\theta_2)^4 + 6\theta_1\theta_2(\theta_1+\theta_2)^5 + \\ \theta_1(\theta_2^4+6)(\theta_1+\theta_2)^6 \end{bmatrix}}{(\theta_1^4+6)(\theta_2^4+6)(\theta_1+\theta_2)^7} \tag{5}$$

If we have two ordered random samples representing strength (X1,X2...Xn) and stress (Y1,Y2,....,Ym) having sizes n and m respectively, following Pranav distribution with parameters θ_1 and θ_2, respectively. A technique for figuring out a model's parameter values is called maximum likelihood estimation. The parameter values were selected to maximise the likelihood that the model's described process resulted in the observed data. The optimal distribution for a collection of data is chosen using the maximum likelihood estimate (MLE). A reliable method for estimating parameters is maximum likelihood. So, a variety of estimation scenarios can apply maximum likelihood estimations. With those parameter values, the likelihood of those observed outcomes is the same as the probability of those observed events. Thus, the likelihood function for the combined random sample can be calculated:

$$L = \prod_{i=1}^{n} \frac{\theta_1^4}{\theta_1^4+6}(\theta_1 + x_i^3)\, e^{-\theta_1 x_i^3} \prod_{j=1}^{n} \frac{\theta_2^4}{\theta_2^4+6}(\theta_2 + y_j^3) e^{-\theta_2 y_j^3} \tag{6}$$

the log likelihood is

$$l = logL = 4nlog\theta_1 - nlog(\theta_1^4+6) + \sum_{i=0}^{n}(log(\theta_1 + x_i^3)) - \theta_1 \sum_{i=0}^{n} x_i$$
$$+ 4mlog\theta_2 - mlog(\theta_2^4+6) + \sum_{j=1}^{m}(log(\theta_2 + y_j^3)) - \theta_2 \sum_{j=1}^{m} y_j \tag{7}$$

The solution of the following non-linear equations yield the MLE of the parameters parameters θ_1 and θ_2. Differentiating (7) with respect to parameters θ_1 and θ_2, we obtain

$$\frac{\partial l}{\partial \theta_1} = \frac{4n}{\theta_1} - \frac{4n\theta_1^3}{(\theta_1^4)+6} + \sum_{i=0}^{n} \frac{1}{\theta_1 + x_i^3} - \sum_{i=0}^{n} x_i \tag{8}$$

$$\frac{\partial l}{\partial \theta_2} = \frac{4m}{\theta_2} - \frac{4m\theta_2^3}{(\theta_2^4)+6} + \sum_{j=0}^{m} \frac{1}{\theta_2 + x_j^3} - \sum_{j=0}^{m} y_j \tag{9}$$

The second partial derivatives of (7) with respect to parameters θ_1 and θ_2, are

$$\frac{\partial l^2}{\partial \theta_1^2} = \frac{-4n}{\theta_1^2} - \frac{4n\theta_1^2(18-\theta_1^4)}{(\theta_1^4+6)^2} - \sum_{i=0}^{n} \frac{1}{(\theta_1 + x_i^3)^2} \tag{10}$$

$$\frac{\partial l^2}{\partial \theta_2^2} = \frac{-4m}{\theta_2^2} - \frac{4m\theta_2^2(18-\theta_2^4)}{(\theta_2^4+6)^2} - \sum_{j=0}^{m} \frac{1}{(\theta_2 + y_j^3)^2} \tag{11}$$

MLE of R is obtained as

$$\hat{R} = 1 - \frac{\hat{\theta}_1 \begin{bmatrix} 360\hat{\theta}_1(\hat{\theta}_2{}^2 + 1080\hat{\theta}_2{}^3 + 144\hat{\theta}_2(\hat{\theta}_1+\hat{\theta}_2)^2 + \\ 6(\hat{\theta}_1\hat{\theta}_2{}^3 + \hat{\theta}_2{}^4 + 6)(\hat{\theta}_1+\hat{\theta}_2)^3 + 6(\hat{\theta}_1\hat{\theta}_2)^2(\hat{\theta}_1+\hat{\theta}_2)^4 + \\ 6\hat{\theta}_1\hat{\theta}_2(\hat{\theta}_1+\hat{\theta}_2)^5 + \hat{\theta}_1(\hat{\theta}_2{}^4+6)(\hat{\theta}_1+\hat{\theta}_2)^6 \end{bmatrix}}{(\hat{\theta}_1{}^4+6)(\hat{\theta}_2{}^4+6)(\hat{\theta}_1+\hat{\theta}_2)^7} \tag{12}$$

Where $\hat{\theta_1}$ and $\hat{\theta_2}$ are the maximum likelihood estimators of θ_1 and θ_2 respectively. This is used in estimation of stress strength for given data.

3. Asymptotic Distribution and Confidence Interval

The asymptotic distribution and confidence interval of the MLE of R are obtained in this section. To find an asymptotic variance of the MLE $\hat{R}^{ML}$, let us denote the Fisher information matrix of $\theta = (\theta_1, \theta_2)$ as $I(\theta) = [I_ij(\theta); i, j = 1, 2]$, i.e.,

$$I(\theta) = E \begin{bmatrix} \frac{-\partial^2 l}{\partial \theta_1^2} & \frac{-\partial^2 l}{\partial \theta_1 \partial \theta_2} \\ \frac{-\partial^2 l}{\partial \theta_2 \partial \theta_1} & \frac{\partial^2 l}{\partial \theta_2^2} \end{bmatrix} \tag{13}$$

To establish normality assumption we define

$$d(\theta) = (\frac{\partial R}{\partial \theta_1}, \frac{\partial R}{\partial \theta_2})' \tag{14}$$

$$= (d_1, d_2)' \tag{15}$$

R is in the form

$$R = 1 - \frac{U}{V} P \tag{16}$$

Hence

$$\frac{\partial R}{\partial \theta_i} = - \left[(\frac{U}{V})' P + P' \frac{U}{V} \right], i = 1, 2 \tag{17}$$

$$U = \theta_1 \tag{18}$$

$$V = (\theta_1^4 + 6)(\theta_2^4 + 6)(\theta_1 + \theta_2)^7 \tag{19}$$

$$P = 360\hat{\theta_1}(\hat{\theta_2}^2 + 1080\hat{\theta_2}^3 + 144\hat{\theta_2}(\hat{\theta_1} + \hat{\theta_2})^2 + 6(\hat{\theta_1}\hat{\theta_2}^3 + \hat{\theta_2}^4 + 6)(\hat{\theta_1} + \hat{\theta_2})^3 + 6(\hat{\theta_1}\hat{\theta_2})^2(\hat{\theta_1} + \hat{\theta_2})^4 + 6\hat{\theta_1}\hat{\theta_2}(\hat{\theta_1} + \hat{\theta_2})^5 + \hat{\theta_1}(\hat{\theta_2}^4 + 6)(\hat{\theta_1} + \hat{\theta_2})^6 \tag{20}$$

The partial derivatives of $\frac{U}{V}$ and P with respect to θ_1 is

$$\frac{\partial P}{\partial \theta_1} = 360\theta_2^2 + 288\theta_2(\theta_1 + \theta_2) + 18\theta_1\theta_2^3 + (\theta_1 + \theta_2)^2 + 6\theta_2^3(\theta_1 + \theta_2)^3 + 18\theta_2(\theta_1 + \theta_2)^2 + 18(\theta_1 + \theta_2)^2 + 24(\theta_1\theta_2)^2(\theta_1 + \theta_2)^3 + 12\theta_1\theta_2^2(\theta_1 + \theta_2)^4 + 30\theta_1\theta_2(\theta_1 + \theta_2)^4 + 6\theta_2(\theta_1 + \theta_2)^5 + 6\theta_1(\theta_2^4 + 6)(\theta_1 + \theta_2)^5 + (\theta_2^4 + 6)(\theta_1 + \theta_2)^6 \tag{21}$$

$$(\frac{U}{V})' = \frac{\begin{bmatrix} (\theta_1^4 + 6)(\theta_2^4 + 6)(\theta_1 + \theta_2)^7 - \\ 7\theta_1(\theta_1^4 + 6)(\theta_2^4 + 6)(\theta_1 + \theta_2)^6 - \\ 4\theta_1^4(\theta_2^4 + 6)(\theta_1 + \theta_2)^7 \end{bmatrix}}{[(\theta_1^4 + 6)(\theta_2^4 + 6)(\theta_1 + \theta_2)^7]^2} \tag{22}$$

Then partial derivative of R with respect to θ_1 becomes

$$\frac{\partial R}{\partial \theta_1} = -\frac{\begin{bmatrix} -7\theta_1(\theta_2^4+6)(\theta_2^4+6)(\theta_1+\theta_2)^6 - 4\theta_1^4(\theta_2^4+6)(\theta_1+\theta_2)^7 + \\ (\theta_1^4+6)(\theta_2^4+6)(\theta_1+\theta_2)^7 \end{bmatrix}}{((\theta_1^4+6)(\theta_2^4+6)(\theta_1+\theta_2)^7)^2}$$
$$\times \begin{bmatrix} 360\theta_1\theta_2^2 + 1080\theta_2^3 + 144\theta_2(\theta_1+\theta_2)^2 + \\ 6(\theta_1\theta_2^3+\theta_2^4+6)(\theta_1+\theta_2)^3 + 6(\theta_1\theta_2)^2(\theta_1+\theta_2)^4 + \\ 6(\theta_1\theta_2)^2(\theta_1+\theta_2)^4 + 6\theta_1\theta_2(\theta_1+\theta_2)^5 + \theta_1(\theta_2^4+6)(\theta_1+\theta_2)^6 \end{bmatrix}$$
$$+ \begin{bmatrix} 360\theta_2^2 + 288\theta_2(\theta_1+\theta_2) + 18\theta_1\theta_2^3(\theta_1+\theta_2)^2 6\theta_2^3(\theta_1+\theta_2)^3 + \\ 18\theta_2(\theta_1+\theta_2)^2 + 18(\theta_1+\theta_2)^2 + 24(\theta_1\theta_2)^2(\theta_1+\theta_2)^3 \\ +12\theta_1\theta_2^2(\theta_1+\theta_2)^4 + 30\theta_1\theta_2(\theta_1+\theta_2)^4 + \\ 6\theta_2(\theta_1+\theta_2)^5 + 6\theta_1(\theta_2^4+6)(\theta_1+\theta_2)^5 + (\theta_2^4+6)(\theta_1+\theta_2)^6 \end{bmatrix}$$
$$\times \frac{\theta_1}{(\theta_1^4+6)(\theta_2^4+6)(\theta_1+\theta_2)^7} \tag{23}$$

The partial derivatives of $\frac{U}{V}$ and P with respect to θ_2 is

$$\begin{aligned} \frac{\partial P}{\partial \theta_2} &= 720\theta_1\theta_2 + 3240\theta_2^2 + 288\theta_2(\theta_1+\theta_2) + 144(\theta_1+\theta_2)^2 \\ &+ 18\theta_1\theta_2^3(\theta_1+\theta_2)^2 + 18\theta_1\theta_2^2(\theta_1+\theta_2)^3 + 18\theta_2^4(\theta_1+\theta_2)^2 + 24\theta_2^3(\theta_1+\theta_2)^3 \\ &+ 18(\theta_1+\theta_2)^2 + 24(\theta_1\theta_2)^2(\theta_1+\theta_2)^3 + 12\theta_1^2\theta_2(\theta_1+\theta_2)^4 + 30\theta_1\theta_2(\theta_1+\theta_2)^4 \\ &+ 6\theta_1(\theta_1+\theta_2)^5 + 6\theta_1(\theta_2^4+6)(\theta_1+\theta_2)^5 + 4\theta_1\theta_2^3(\theta_1+\theta_2)^6 \end{aligned} \tag{24}$$

$$(\frac{U}{V})' = -\frac{\left[-7\theta_1(\theta_1^4+6)(\theta_2^4+6)(\theta_1+\theta_2)^6 - 4\theta_1\theta_2^3(\theta_1^4+6)(\theta_1+\theta_2)^7\right]}{((\theta_1^4+6)(\theta_2^4+6)(\theta_1+\theta_2)^7)^2} \tag{25}$$

Then the partial derivative of R with respect to θ_2 becomes

$$\frac{\partial R}{\partial \theta_2} = -\frac{\left[-7\theta_1(\theta_1^4+6)(\theta_2^4+6)(\theta_1+\theta_2)^6 - 4\theta_1\theta_2^3(\theta_1^4+6)(\theta_1+\theta_2)^7\right]}{((\theta_1^4+6)(\theta_2^4+6)(\theta_1+\theta_2)^7)^2}$$
$$\times \begin{bmatrix} 360\theta_1\theta_2^2 + 1080\theta_2^3 + 144\theta_2(\theta_1+\theta_2)^2 + \\ 6(\theta_1\theta_2^3+\theta_2^4+6)(\theta_1+\theta_2)^3 + 6(\theta_1\theta_2)^2(\theta_1+\theta_2)^4 + \\ 6(\theta_1\theta_2)^2(\theta_1+\theta_2)^4 + 6\theta_1\theta_2(\theta_1+\theta_2)^5 + \theta_1(\theta_2^4+6)(\theta_1+\theta_2)^6 \end{bmatrix}$$
$$+ \begin{bmatrix} 720\theta_1\theta_2 + 3240\theta_2^2 + 288\theta_2(\theta_1+\theta_2) + 144(\theta_1+\theta_2)^2 + \\ 18\theta_1\theta_2^3(\theta_1+\theta_2)^2 + 18\theta_1\theta_2^2(\theta_1+\theta_2)^3 + 18\theta_2^4(\theta_1+\theta_2)^2 + 24\theta_2^3(\theta_1+\theta_2)^3 + \\ 18(\theta_1+\theta_2)^2 + 24(\theta_1\theta_2)^2(\theta_1+\theta_2)^3 + 12\theta_1^2\theta_2(\theta_1+\theta_2^4) + 30\theta_1\theta_2(\theta_1+\theta_2)^4 + \\ 6\theta_1(\theta_1+\theta_2)^5 + 6\theta_1(\theta_2^4+6)(\theta_1+\theta_2)^5 + 4\theta_1\theta_2^3(\theta_1+\theta_2)^6 \end{bmatrix}$$
$$\times \frac{\theta_1}{(\theta_1^4+6)(\theta_2^4+6)(\theta_1+\theta_2)^7} \tag{26}$$

We obtain the asymptotic distribution of $\hat{R}^{ML}$ as

$$\sqrt{n+m}(\hat{R}^{ML} - R) \to N(0, d'(\theta)I^{-1}(\theta)d(\theta)) \tag{27}$$

$$AV(\hat{R}^{ML}) = \frac{1}{n+m}d'(\theta)I^{-1}(\theta)d(\theta) \tag{28}$$

$$i.e, AV(\hat{R}^{ML}) = V(\hat{\theta_1})d_1^2 + V(\hat{\theta_2})d_2^2 + 2d_1d_2Cov(\hat{\theta_1}, \hat{\theta_2}) \tag{29}$$

Asymptotic $100(1-\alpha)$ percentage confidence interval for R

$$\hat{R}^{ML} \pm Z_{\alpha/2}\sqrt{AV(\hat{R}^{ML})} \tag{30}$$

4. Bootstrap Confidence Interval

In this section, we use confidence intervals based on the parametric percentile bootstrap methods (we call it from now on as Boot-p) based, [10]. Bootstrapping is a technique for estimating the variability in a statistic by sampling with replacement from observed data. Permutation tests are a type of re-sampling that is linked to re-sampling. The bootstrap is frequently used to evaluate the accuracy of an estimate based on a sample of data from a larger population. The bootstrap's main advantage is that it allows statisticians to construct confidence intervals on parameters without making irrational assumptions. This was one of the first of many discoveries in computational statistics, which has now become the standard method for practically all work. It creates multiple re-samples (with replacement) from a single set of observations, and computes the effect size of interest on each of these re-samples. To estimate confidence intervals of R in this methods, the following steps are used.

1. Estimate θ, say $\hat{\theta}$, from the sample using MLE method.
2. Generate a bootstrap sample using $\hat{\theta}$. Using these bootstrap sample obtain the bootstrap estimate of θ, say $\hat{\theta}*$ and compute the bootstrap estimate of R.
3. Repeat Step [2] N-BOOT times to get the parametric bootstrap estimates of R
4. Let $\widehat{CDF} = P(\hat{\theta}* \leq x)$, be the cumulative distribution function of $\hat{R}$. Define $\hat{\theta}_{Boot-p}(x) = \widehat{CDF}^{-1}(x)$ for a given x. The approximate $(100-\alpha)\%$ confidence interval for θ is given by

$$(\hat{\theta}_{BOOT-P}(\alpha/2), \hat{\theta}_{BOOT-P}(1-(\alpha/2))) \tag{31}$$

5. Simulation Study

To measure the efficacy of R's estimators, we provide some results based on the inversion method in this section. According to its definition, data simulation is the process of using a significant amount of data to replicate or simulate real-world settings in order to determine the optimal course of action, predict future events, or test a model. The many forms of data simulations are numerous. In simulation statistics, artificially generated data are used to test a hypothesis or statistical method. Every time a new statistical method is created or used, certain assumptions need to be verified. Simulated data is used by statisticians to test their theories. For this purpose, we have generated 1000 samples from independent Pranav(θ_1) and Pranav(θ_2) distributions. We considered sets of parameter values (1) and (1.99) parameter values. The bias and the mean square error (MSE) of the parameter estimates are calculated. In Table 5.1, the average biases, mean squared errors (MSE) and confidence intervals of the estimates of R is given.

Table 1: *Simulation Results*

(n,m)	R(ML)	Bias	MSE	AS(CI)	BT(CI)
(7,7)	0.8982136	0.1017854	0.01036027	0.8783155,0.9181117	0.8947541,0.8999972
(15,15)	0.8982252	0.1017738	0.0103579	0.8782696,0.9181808	0.8966189,0.8994737
(20,20)	0.8982344	0.1017646	0.01035603	0.8783453,0.9181235	0.8965753,0.8999412
(22,22)	0.8985076	0.1014914	0.0103005	0.8787993,0.9182159	0.8969416,0.8993731
(25,25)	0.8985383	0.1014607	0.01029428	0.8789089,0.9181677	0.8973348,0.898984

From the simulation results, it is observed that as the sample size (n, m) increases the biases and the MSE decrease. Thus the consistency properties of all the methods are verified.

6. Data Analysis

In this section we present a data analysis of the strength data reported by Badar and Priest(1982). The data represent the strength data measured in GPA, for single carbon fibers and impregnated 1000-carbon fiber tows. Single fibers were tested under tension at gauge lengths of 1, 10, 20 and 50 mm. Impregnated tows of 1000 fibers were tested at gauge lengths of 20, 50, 150 and 300 mm. It is already observed that the Weibull model does not work well in this case. Surles and Padgett[16], [17] and Raqab and Kundu[12] observed that generalized Rayleigh works quite well for these strength data. For illustrative purposes we are also considering the same transformed data sets as it was considered by Raqab and Kundu [9], the single fibers of 20 mm (Data Set I) and 10 mm (Data Set II) in gauge lengths with sample sizes 69 and 63 respectively. They are presented below:
Data Set I: 0.312 0.314 0.479 0.552 0.700 0.803 0.861 0.865 0.944 0.958 0.966 0.997 1.006 1.021 1.027 1.055 1.063 1.098 1.140 1.179 1.224 1.240 1.253 1.270 1.272 1.274 1.301 1.301 1.359 1.382 1.382 1.426 1.434 1.435 1.478 1.490 1.511 1.514 1.535 1.554 1.566 1.570 1.586 1.629 1.633 1.642 1.648 1.684 1.697 1.726 1.770 1.773 1.800 1.809 1.818 1.821 1.848 1.880 1.954 2.012 2.067 2.084 2.090 2.096 2.128 2.233 2.433 2.585 2.585
Data Set II: 0.101 0.332 0.403 0.428 0.457 0.550 0.561 0.596 0.597 0.645 0.654 0.674 0.718 0.722 0.725 0.732 0.775 0.814 0.816 0.818 0.824 0.859 0.875 0.938 0.940 1.056 1.117 1.128 1.137 1.137 1.177 1.196 1.230 1.325 1.339 1.345 1.420 1.423 1.435 1.443 1.464 1.472 1.494 1.532 1.546 1.577 1.608 1.635 1.693 1.701 1.737 1.754 1.762 1.828 2.052 2.071 2.086 2.171 2.224 2.227 2.425 2.595 3.220
These data were first used by Badar and Priest(1982) and later used by Raqab and Kundu[13] and Hassan and Kumaraswamy [7].

Table 2: *Data Analysis Results*

Length(in mm)	MLE	K-S statistic	P-value
10	1.596362	0.62319	0.7571
20	1.715981	0.79365	0.4375

Table 6.1 gives the result of the goodness of fit test. Maximum likelihood estimates are 1.596362 and 1.715981.The estimated value of the stress strength reliability using these estimates is 0.8850866.The 95% asymptotic confidence interval for R is (0.8530591,0.9171141) and 95% bootstrap confidence interval for R is (0.8748553 ,0.8895613).

7. Conclusion

In this paper, we considered the problem of estimating stress strength reliability using Pranav distribution. The maximum likelihood estimate of stress strength reliability, $\hat{R}$ is obtained. Also, asymptotic $100(1-\alpha)\%$ confidence interval for the reliability parameter is computed. Bootstrap confidence interval for the reliability parameter is also computed. When the sample size is increased, mean square error caused by the estimates comes nearer to zero by extensive simulation. Finally, real data sets are analyzed

References

[1] KE Ahmad, ME Fakhry, and ZF Jaheen. Empirical bayes estimation of $p(y < x)$ and characterizations of burr-type x model. *Journal of Statistical Planning and Inference*, 64(2):297–308, 1997.

[2] Adnan M Awad and Mohamed K Gharraf. Estimation of $p(y < x)$ in the burr case: A comparative study. *Communications in Statistics-Simulation and Computation*, 15(2):389–403, 1986.

[3] James D Church and Bernard Harris. The estimation of reliability from stress-strength relationships. *Technometrics*, 12(1):49–54, 1970.

[4] Kenneth Constantine, Siu-Keung Tse, and Marvin Karson. Estimation of p (y< x) in the gamma case. *Communications in Statistics-Simulation and Computation*, 15(2):365–388, 1986.

[5] F Downton. The estimation of $pr(y < x)$ in the normal case. *Technometrics*, 15(3):551–558, 1973.

[6] Rameshwar D Gupta and Debasis Kundu. Closeness of gamma and generalized exponential distribution. *Communications in statistics-theory and methods*, 32(4):705–721, 2003.

[7] Marwa KH Hassan. The kumaraswamy exponential-weibull distribution and its application in reliability. *International Journal of Reliability and Applications*, 19(2):95–108, 2018.

[8] Shukla KK. Pranav distribution with properties and its applications. *Biom Biostat Int J*, 7(3):244–254, 2018.

[9] Debasis Kundu and Rameshwar D Gupta. Estimation of $p[y < x]$ for generalized exponential distribution. *Metrika*, 61(3):291–308, 2005.

[10] Debasis Kundu, Nandini Kannan, and N Balakrishnan. Analysis of progressively censored competing risks data. *Handbook of statistics*, 23:331–348, 2003.

[11] Debasis Kundu and Mohammad Z Raqab. Estimation of $r = p(y < x)$ for three-parameter weibull distribution. *Statistics & Probability Letters*, 79(17):1839–1846, 2009.

[12] Mohammad Z Raqab and Debasis Kundu. Comparison of different estimators of $p[y < x]$ for a scaled burr type x distribution. *Communications in Statistics"Simulation and Computation®*, 34(2):465–483, 2005.

[13] Mohammad Z Raqab, Mohamed T Madi, and Debasis Kundu. Estimation of $p(y < x)$ for the three-parameter generalized exponential distribution. *Communications in Statistics"Theory and Methods*, 37(18):2854–2864, 2008.

[14] Buğra Saraçoğlu, Ismail Kinaci, and Debasis Kundu. On estimation of $r = p(y < x)$ for exponential distribution under progressive type-ii censoring. *Journal of Statistical Computation and Simulation*, 82(5):729–744, 2012.

[15] Fatemeh Shahsanaei and Alireza Daneshkhah. Estimation of stress-strength model in the generalized linear failure rate distribution. *arXiv preprint arXiv:1312.0401*, 2013.

[16] JG Surles and WJ Padgett. Inference for $p(y < x)$ in the burr type x model. *Journal of Applied Statistical Science*, 7(4):225–238, 1998.

[17] JG Surles and WJ Padgett. Inference for reliability and stress-strength for a scaled burr type x distribution. *Lifetime data analysis*, 7:187–200, 2001.

[18] Akhila K Varghese and VM Chacko. Estimation of stress-strength reliability for akash distribution. *Reliability: Theory & Applications*, 17(3 (69)):52–58, 2022.